

CS 381 Solutions to Homework 7

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- 2 (a) $A \cap B$
- (b) $A \cap \overline{B}$
- (c) $A \cup B$
- (d) $\overline{A} \cup \overline{B}$

8 (a) For any element x , $x \in A \cup A \leftrightarrow (x \in A \vee x \in A) \leftrightarrow x \in A$
Hence $A \cup A = A$.

(b) For any element x , $x \in A \cap A \leftrightarrow (x \in A \wedge x \in A) \leftrightarrow x \in A$
Hence $A \cap A = A$.

14. $A = \{1, 3, 5, 6, 7, 8, 9\}$ $B = \{2, 3, 6, 9, 10\}$

16 (e) For any element x , $x \in A \cup (B - A) \leftrightarrow x \in A \vee (x \in B \wedge \neg x \in A)$
 $\leftrightarrow (x \in A \vee x \in B) \wedge (x \in A \vee \neg x \in A)$
 $\leftrightarrow (x \in A \vee x \in B) \leftrightarrow x \in A \cup B$
Hence $A \cup (B - A) = A \cup B$.

$$\begin{aligned} 24. (A - C) - (B - C) &= (A \cap \overline{C}) \cap \overline{(B - C)} \\ &= (A \cap \overline{C}) \cap \overline{(B \cap \overline{C})} \\ &= (A \cap \overline{C}) \cap (\overline{B} \cup C) \\ &= (A \cap \overline{C} \cap \overline{B}) \cup (A \cap C \cap \overline{C}) \\ &= A \cap \overline{B} \cap \overline{C} \\ &= (A \cap \overline{B}) \cap \overline{C} = (A - B) - C \end{aligned}$$

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24 (a) Basis Clause: $1 \in \text{Odd}$

Inductive Clause: For any integer x , if $x \in \text{Odd}$, then $x + 2 \in \text{Odd}$.

Extremal Clause: Nothing is in *Odd* unless it is obtained by the above two clauses.

(b) Let T denote the set of positive integer powers of 3.

Basis Clause: $3 \in T$.

Inductive Clause: For any integer x , if $x \in T$, then $3x \in T$.

Extremal Clause: Nothing is in T unless it is obtained by the above two clauses.

26 (a) $(0,0)$, $(2,3)$, $(3,2)$, $(4,6)$, $(5,5)$, $(6,4)$, $(6,9)$, $(7,8)$, $(8,7)$, $(8,12)$, $(9,6)$, $(9,11)$, $(10,10)$, $(11,9)$, $(12,8)$.