

## CS 381 Solutions to Homework 9

Q 1. Textbook p. 581:

2. (a)  $\langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 1, 5 \rangle, \langle 1, 6 \rangle, \langle 2, 2 \rangle, \langle 2, 4 \rangle, \langle 2, 6 \rangle, \langle 3, 3 \rangle, \langle 3, 6 \rangle, \langle 4, 4 \rangle, \langle 5, 5 \rangle, \langle 6, 6 \rangle$

Q 2. Let  $R$  denote the relation to be defined.

Basis Clause:  $\langle 0, 0 \rangle \in R$

Inductive Clause: If  $\langle x, y \rangle \in R$ , then  $\langle x + 1, y + 3 \rangle \in R$ .

Extremal Clause: Nothing is in  $R$  unless it is obtained from the Basis and Inductive Clauses.

Q3. Every subset of  $\{\langle 0, 0 \rangle, \langle 0, 2 \rangle, \langle 2, 0 \rangle, \langle 2, 2 \rangle\}$ , including the empty set, is a binary relation on  $\{0, 2\}$ . There are 16 of them altogether.

Q 4. Let  $A$  denote the set of cardinality  $n$ . Then a unary relation on  $A$  is a set of 1-tuples of elements of  $A$ , that is a set of  $\langle i \rangle$ 's for elements  $i$  of  $A$ . Hence the number of binary relations on  $A$  is equal to the number of subsets of  $A$ . Since a set  $B$  has  $2^{|B|}$  subsets in general, the number of unary relations on  $A$  is equal to  $2^n$ .