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OPTIMIZATION OF GEOMETRICALLY TRIMMED B-SPLINE SURFACES

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ABSTRACT

Unlike the visual trimming of B-spline surfaces, which hides unwanted portions in rendering, the geometric trimming approach provides a mathematically clean representation without redundancy. However, the process may lead to significant deviation from the corresponding portion on the original surface. Optimization is required to minimize approximation errors and obtain higher accuracy.

In this paper, we describe the application of a novel global optimization method, so-called hybrid Parallel Tempering (PT) and Simulated Annealing (SA) method, for the minimization of B-spline surface representation errors. The high degree of freedom within the configuration of B-spline surfaces as well as the "rugged" landscapes of objective functions complicate the error minimization process. The hybrid PT/SA method, which is an effective algorithm to overcome the slow convergence, waiting dilemma, and initial value sensitivity, is a good candidate for optimizing geometrically trimmed B-spline surfaces. Examples of application to geometrically trimmed wing components are presented and discussed. Our preliminary results confirm our expectation.

INTRODUCTION

In most of the literature, the term "trimming" of B-spline surfaces refers to the visual trimming, which defines a trimmed surface by using the original surface and a trimming curve. When the trimmed surface is rendered, the original surface is tessellated so that it shows only the wanted portions. In contrast, geometric trimming creates new surfaces. Intersections are obtained to restrain the sampling points [1]. Then surface points are sampled on the retained part and re-interpolated into new surfaces [2, 3, 4]. In this paper, trimming refers to geometric trimming for the sake of brevity. An exact trimming is precluded due to the remarkably high degree of their intersections [5]. Depending on the selection of the interpolation points on the original surface, the trimming errors may vary greatly.

The B-spline curve/surface shape modification problem has been addressed through optimization. Ferguson and Jones proposed methods to control curvature by formulating a constrained nonlinear optimization problem using the control points of B-splines in nonrational form as variables [6]. Moreton and Sequin studied the application of nonlinear optimization techniques to minimize a fairness functional based on the variation of curvature [7]. Hohenberger and Reuding investigated the possibilities of entering the weights in an automatic fairing process [8]. Laurent-Gengoux and Mekhilef investigated the optimization of a NURBS representation by using Polak-Ribiere technique with some improvement [9]. The Quasi-Newton BFGS method has been applied to optimize NURBS represented wing profiles by Trepanier, etc [10]. Most prior work focuses on curve shape optimization and local optimization methods due to the high dimensionality and complexity of the surface optimization problem.

In this paper, trimmed B-spline surfaces are optimized by the hybrid PT/SA method, which has been applied to molecular biology and has shown promising results in solving complex, high-dimension problems and overcoming slow convergence [11]. The optimization can be further applied to the general surface fitting problem, not limited to trimmed surfaces.

This paper is organized as follows. First, geometric trimming is explained and the trimming errors are defined. Then the hybrid PT/SA method is described and the implementation to B-spline surface shape optimization is detailed. Finally, results on trimmed wing surfaces are presented and analyzed.

DEFINITION OF A NURBS SURFACE

A non-uniform rational B-spline (NURBS) surface is defined by:

$$S(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}P_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}}$$

The $P_{i,j}$ are control points, the $w_{i,j}$ are the weights, and the $N_{i,p}(u)$ and $N_{j,q}(v)$ are the B-spline basis functions defined on the knot sequences $U = \{u_0, u_1, ..., u_{n+p+1}\}$ and $V = \{v_0, v_1, ..., v_{m+q+1}\}$, where

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u < u_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

p is the degree in the u direction and q is the degree in v direction. $N_{j,q}(v)$ is defined similarly to $N_{i,p}(u)$. If all the weights are the same, it refers to non-rational B-spline surfaces. Since weights are difficult to visualize and very little is known on setting weights, most often all weights are set to 1.

GEOMETRIC TRIMMING

A new surface is created by re-interpolating the surface points in the wanted portion of the original surface. When a surface is trimmed by an open trimming curve whose two end points lie on the opposite boundary in the u-v space, the surface is divided into two regions by the trimming curve. The rule is set so that the wanted portion is on the left side when walking along the trimming curve (Figure 1).

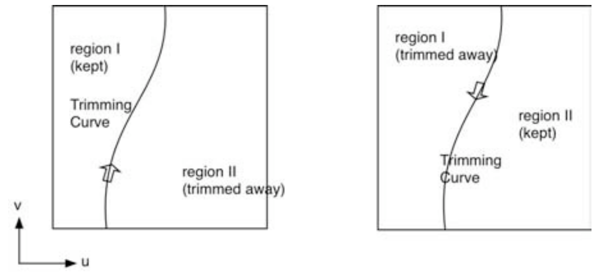


Figure 1 Illustration of trimming curves and regions

Due to the properties of tensor-product surfaces, the number of interpolation points on each isoparametric curve must be the same. Let (u_{ltrim}, v_l) be the trimming curve point on the lth v isoparametric curve. The sampling points are thus selected uniformly in the wanted portion. The sampling points are $(u_k, v_l) = (u_{ltrim} \frac{k}{N^* - 1}, v_l)$ for region I in figure 1 and $(u_k, v_l) = \left(u_{ltrim} + (u_{\max} - u_{ltrim}) \frac{k}{N^* - 1}, v_l \right)$ for region II, where N^* is the number of points in the new parametric space on each v isoparametric curve. N^* can be any number as long as it satisfies the minimum number of points required by the surface degree and interpolation method. Increasing the number of points forces a closer adherence to the original shape. The determination of isoparametric curves also affects the results. If the original surface is created by interpolation and we have the knowledge of the original interpolation, the knowledge can be applied to trimming sufficiently. Using the parameters of the original interpolation points for sampling the new surface points and computing parameters by the same parameterization method provides high accuracy. However, in many cases, the knowledge of how the original surface is created is unknown. Uniformly distributed isoparametric curves are selected.

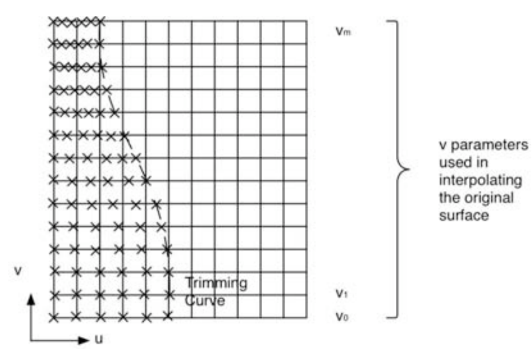
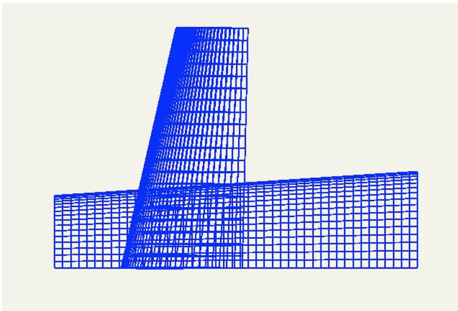
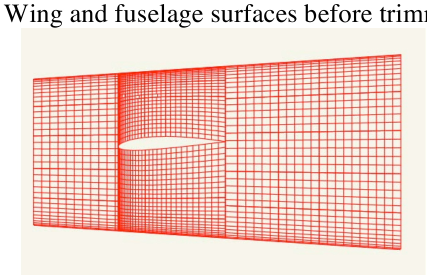


Figure 2 Trimming by an open trimming curve

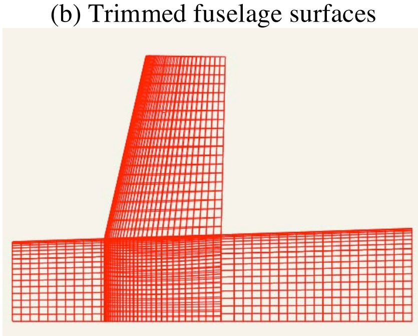
Surfaces trimmed by other types of trimming curves require surface subdivision and conversion of trimming curves [2]. Figure 3 shows geometrically trimmed wing and fuselage surfaces.



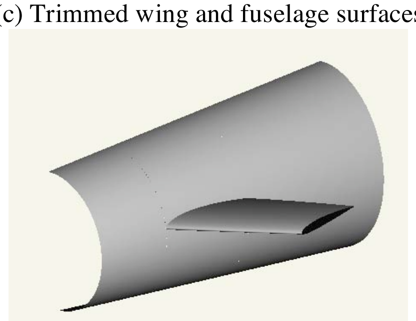
(a) Wing and fuselage surfaces before trimming



(b) Trimmed fuselage surfaces



(c) Trimmed wing and fuselage surfaces



(d) Trimmed wing and fuselage surfaces in shading

Figure 3 Trimming of fuselage and wing surfaces

ERROR DEFINITION

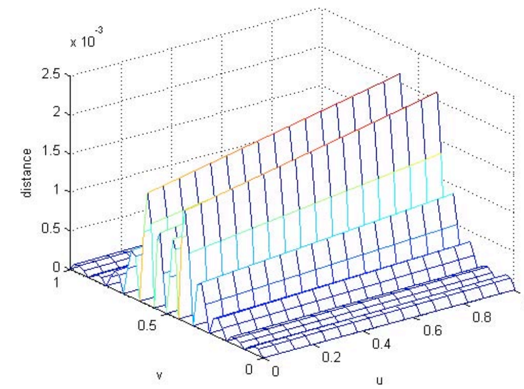
The difference between the original surface and the trimmed one can be evaluated by

$$d_1 = \sum_{i=0}^{r-1} \left| \mathbf{x}_i - \mathbf{x}_i^* \right| \Big/ r$$

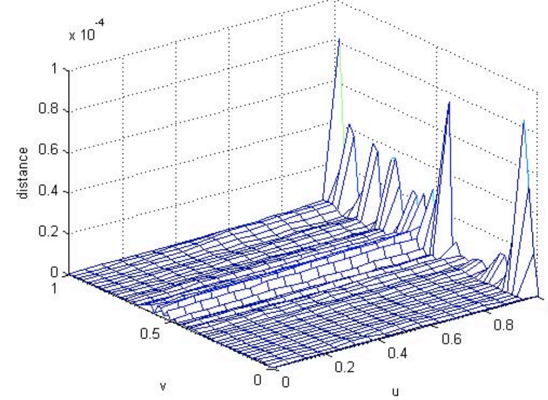
$$d_2 = \sqrt{\sum_{i=0}^{r-1} \left| \mathbf{x}_i - \mathbf{x}_i^* \right|^2} \Big/ r$$

$$d_{\inf} = \max_i \left| \mathbf{x}_i - \mathbf{x}_i^* \right|$$

where $\left| \mathbf{x}_i - \mathbf{x}_i^* \right| = \sqrt{(x_i - x_i^*)^2 + (y_i - y_i^*)^2 + (z_i - z_i^*)^2}$ is the distance between two surface points in the Euclidean space and r is the total number of sampling points. $\mathbf{x}_i$ are the points on the retained portion of the original surface and $\mathbf{x}_i^*$ are the images of $\mathbf{x}_i$ on the trimmed surface. The images are created by projecting points on the original surface to the trimmed surface using the Newton method, which is a classical method to find the closest point on a surface to a given point [12]. The exception is the boundary, where the points are projected to the boundary curve of the trimmed surface instead of the surface. Figure 4 shows the distance between the associated points for the trimmed wing. In Figure 4(a), the isoparametric curves are selected uniformly, while in Figure 4(b), the parameters and parameterization method in interpolating the original surface are used. The d_∞ of (a) is 2.1903e-3 and the d_∞ of (b) is 8.3100e-5. The error in (a) is significantly greater than that in (b). When the construction of the original surface is unknown, a greater error might occur. The hybrid PT/SA is applied to optimize the surface and reduce errors.



(a)



(b)

Figure 4 Distance plots of the surfaces trimmed by not knowing and knowing the interpolation of the original surface

HYBRID PARALLEL TEMPERING/SIMULATED ANNEALING METHOD

The SA method is analogous to metals cooling and annealing [13]. If the liquid is cooled slowly, the atoms are often able to line themselves up and form a pure crystal that is completely ordered. This crystal is the state of minimum energy for this system. The Boltzmann probability distribution states that a system in thermal equilibrium at temperature T has its energy probabilistically distributed among all different energy states E as $\text{Prob}(E) \sim \exp(-E/kT)$. The quantity k is a constant of nature that relates temperature to energy. The system sometimes goes uphill as well as downhill, so that it has a chance to escape from a local energy minimum to find a better one. But the lower the temperature, the smaller probability it goes uphill. It accepts a move if it lowers the energy. Otherwise, the move will be accepted with probability $e^{-(E^{new}-E^{old})/kT}$. Initially, the SA method raises the temperature to a high value, and slowly and gradually reduces it during the optimization procedure to help escape from the local minimums. This method can also be used for other systems by introducing a control parameter, analogous to temperature, and an annealing cooling schedule that describes its gradual reduction.

The PT method, also known as the multiple Markov chain or replica-exchange method, builds up on Markov chains of different temperatures [14]. The main idea of PT is that the replica transition moves between different temperature levels, which enables the system at the low temperature level to escape from local minima and to locate multiple minima by allowing it to switch with the system configuration at higher temperature according to the Metropolis-Hasting rule [15]. The replica moves accelerate the system to reach equilibrium.

The main idea of the hybrid PT/SA method is to apply the PT moves to the SA scheme to reduce the relaxation time to equilibrium when temperature is changed in SA, which significantly improves the converge rate of the optimization process. By combining the PT and SA methods, a configuration is able to switch between low and high temperature levels in the evolving of the optimization process and at each level while all temperatures will gradually cool down to their target temperatures. The moves at high temperature levels intend to search the energy landscape extensively with greater step length while those at the low temperature levels explore the local details.

The hybrid PT/SA method contains two sets of moves. One is to move on the same Markov chain. The acceptance rate, $\min(1, e^{-(E^{new}-E^{old})/kT})$, is defined in the same way as that of the SA method. The other move is the replica transition, which exchanges the configurations on two adjacent Markov chains:

$$x_j^{new} = x_{j+1}^{old}$$

$$x_{j+1}^{new} = x_j^{old}$$

The two adjacent Markov chains are selected randomly and the replica exchange move is accepted with the probability of $\min(1, e^{-(E_{j+1}/kT_j + E_j/kT_{j+1}) + (E_j/kT_j + E_{j+1}/kT_{j+1})})$, where E_j denotes the energy on the current configuration of the *j*th Markov chain.

OBJECTIVE FUNCTION AND VARIABLES

The objective function, which is associated with the "energy" function in the hybrid PT/SA method, is built as a combination of d_2 and d_∞.

$$F(X) = const_2 \times d_2^2 + const_\infty \times d_\infty^2$$

where X is the vector of variables containing the positions of the NURBS control points, weights or knot sequences. The normalization constants $const_2$ and $const_\infty$ are used to obtain the same order for the two terms and adjust the objective function ranges. Using the square of d_2 and d_∞ is for efficiency. At each iteration the objective function is computed based on a significant number of points. Avoiding the square root operation greatly reduces the computation time, and yet reflects the difference between the two surfaces.

The approximation of the surface with a fixed degree and fixed number of control points is investigated. A NURBS surface is determined by control points, weights, and knots. The movement of control points has no constraints in Euclidean space while the weights must stay positive and the knot sequence must be non-decreasing. The dimension of variables can easily go beyond one hundred. A NURBS surface with degree (p,q) and $m \times n$ control points has two knot sequences of $m+p+1$ and $n+q+1$. Since the knots are relative values, it can be normalized on $[0,1]$, the first and the last knots can be fixed in the optimization. If clamped knot sequences are used, the first $p+1$ knots are the same, so are the last $p+1$ knots. This further reduces the number of knot variables. The size of the control point location variables is $3mn$ while that of the weight variables is mn. Compared to the results of modifying only the control point locations or knots, those of modifying only the weights are insignificant. Furthermore, evaluation of surface points of a rational B-spline surface is more time-consuming. Figure 5 compares the results obtained by modifying control point locations and knots. Modifying control point locations yields better results. Setting both the control point locations and knots as variables increases the dimension of variables. As a result, tuning the optimization parameters is more difficult. Therefore, only control point locations are set as variables.

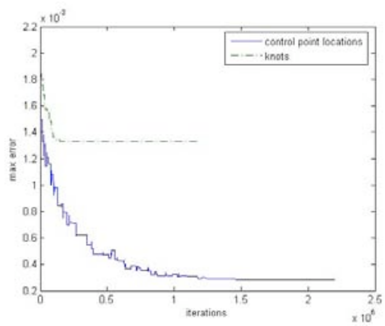


Figure 5 Comparison of maximum error convergence from setting control point locations and knots as variables

IMPLEMENTATION OF HYBRID PT/SA METHOD IN B-SPLINE SURFACE TRIMMING OPTIMIZATION

At each iteration a proposed point (variable vector) is created by a random move from the current point. The scheme of creating the random move is important. The new point can be located far away from the current point to speed up the search process and to avoid getting trapped at a local minimum, or located nearby to focus on the local area. This can be controlled by a step size procedure. If a single variable is moved, the convergence of the system is slow. Allowing every variable to make a small move from the current value is more efficient. The step size in different coordinates should be adjusted to provide efficient moves. For example, the cross-section (airfoil profile) of a wing surface might have greater curvature changes and greater changes of the distances from the surface to the target points, and thus applying step sizes, which allow greater movement in this direction, is more efficient.

Point projection is a computationally costly operation, since every point projection is an optimization process per se [12]. Though good initial values can reduce the convergence time, doing point projection during each iteration slows down the process. However, without doing point projection, the images of target points in the *u-v* parametric space are fixed and the system will be trapped in local minima. Allowing the images of target points in the u-v parametric space to move is critical. An alternative way is to perform point projection every certain number of iterations. To further improve the efficiency, the old values are used as the initial values in point projection.

RESULTS AND DISCUSSION

We apply the hybrid PT/SA method to trimmed wing surfaces. The interpolation points for trimmed surfaces are sampled uniformly in the parametric space of the original surface when the construction of the original surface is unknown. First the method is validated by using target points on a plane.

Validation for a Plane

If the target points are on a plane, the control points are on the plane. The method should be able to move the control points to the plane. We set the 10×10 target points to be uniformly distributed on the plane $z=0$ bounded by lines $x=0, x=1, y=0, y=1$, and a bicubic NURBS surface has 4×4 control points and clamped end knots. Though the solution is not unique, the control points on the boundary should lie on these lines and the z coordinates of all the control points should move toward 0. For different initial surfaces, the optimization process converges to these solutions. Figure 6 gives the distance plots. Figure 7 plots initial and optimized control points. Four temperature levels are applied in this example. The smallest step size is 2e-4. The resulting surface has a maximum distance of 3.99e-5 from the target points.

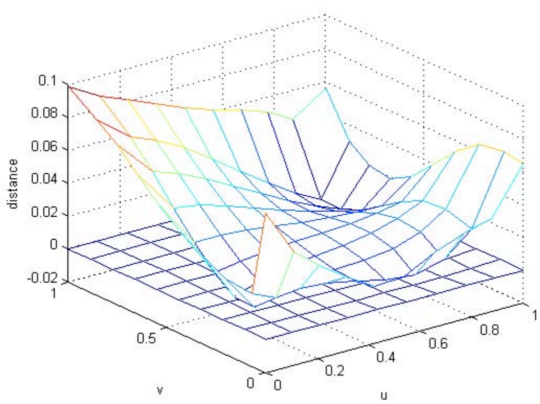
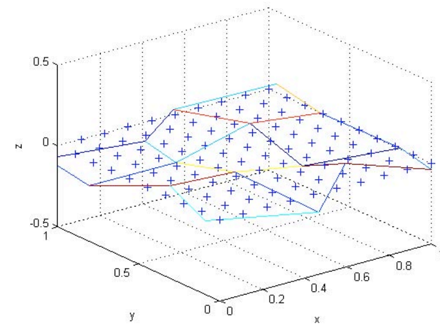
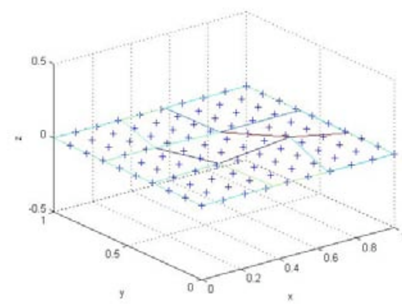


Figure 6 The distance plot of the initial and optimized surfaces (the max error of the initial surface is 9.8887e-2; the max error of the optimized surface is 3.993e-5)



(a) Initial control hull



(b) Optimized control hull

Figure 7 Target points and control hulls

Optimization of Trimmed Wing Surfaces

In this example, the trimmed wing is a bicubic B-spline surface created by interpolating 13×5 surface points (13 along the trimming curve and 5 on each isoparametric curve). Control point locations are set as variables and the problem has a dimension of 195. The target points are 49×17 surface points on the retained part of the original surface (Figure 8).

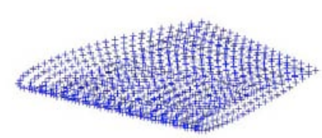


Figure 8 Target points

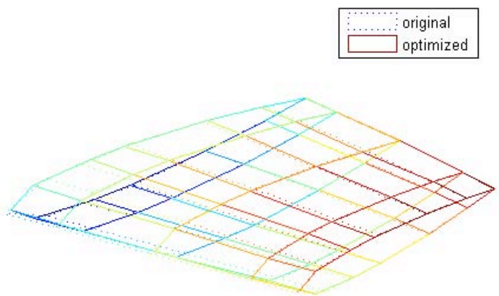


Figure 9 Control hulls of the initial and optimized surfaces

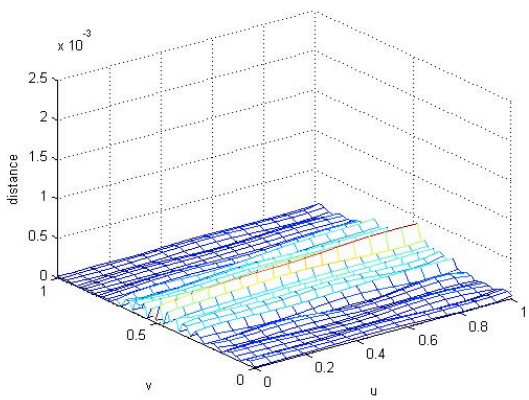


Figure 10 Distance plots after optimization with the same scale of Figure 4(a)

The temperatures are lowered proportionally. Four temperature levels start with 4, 16, 64 and 256, and decrease proportionally by a fraction of 0.997 of (T-1.0) every 1000 steps. The step size varies from 2.0e-6 to 1.28e-4 for x coordinates, 4.0e-7 to 4.58e-5 for y and z coordinates. The optimized surface has the maximum error reduced from 2.0259e-3 to 2.1903e-4 and the distances from the target points distributed evenly. The average error decreases from 6.5665e-4 to 1.0109e-4. The errors are close to the errors of the surface trimmed with prior knowledge of the original surface. Figure 9 and 10 show the control hulls and the distance plots before and after the optimization.

The optimization process is faster on a smaller number of targets points since the complexity of the problem reduces and the time for evaluating the objective function is less. An alternative approach to fit a large number of points is to start with a coarser target point net and refine it progressively. The maximum error of fitting the 49×17 target points by the optimized surface obtained from fitting 25×9 points is 4.0203e-4, compared to that of 2.0259e-3 for the original trimmed surface.

Discussion

For high dimension problems with rough energy landscapes, traditional local optimization methods such as quasi-Newton experiences difficulties in finding good initial values, while the hybrid PT/SA method is not sensitive to initial values. A major problem for using the SA method is the setting of the step size. A big step size misses the details and a small step size leads to slow convergence. Figure 11 compares the convergence of SA and the hybrid PT/SA for fitting 25×9 points of a trimmed wing surface. Four temperature levels are used in the hybrid PT/SA method. The SA with large step size and small step size employ the configuration of the lowest and highest temperature levels of those of the hybrid PT/SA respectively. The number of iterations of the hybrid PT/SA method, which converges to the same level of accuracy, is about 2/3 of that of SA method with small step size. In SA with a big step size, the objective function decreases rapidly at beginning, but is trapped in a local minimum of 4.9 e3. Overall, the hybrid PT/SA approach yields a better convergence rate than both cases in SA. Though these preliminary results are encouraging, studies on setting the parameters are expected to further improve the results.

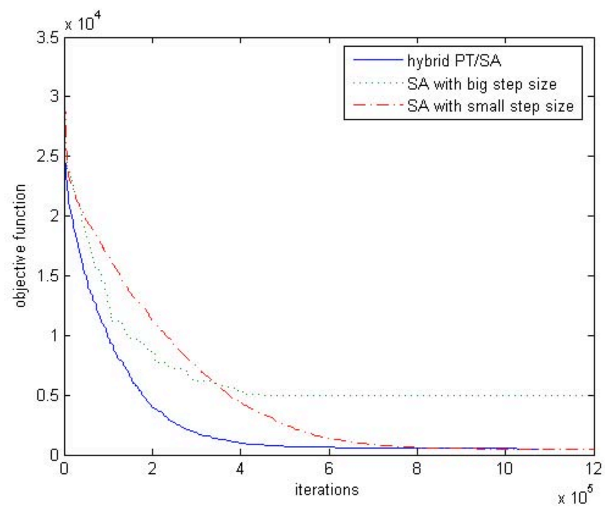


Figure 11 Convergence comparison

CONCLUSIONS

In this work, the hybrid PT/SA method has been shown to be an appropriate method for reducing trimming errors. In our application, the number of the control points and the surface degree are fixed. Control point locations are selected to be variables. Examples are given to reduce geometrically trimmed wing surfaces. In future work, parameter tuning will be explored and trimmed B-spline surfaces and the hybrid PT/SA method will be applied to aerodynamic optimization of aircraft wing components.

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