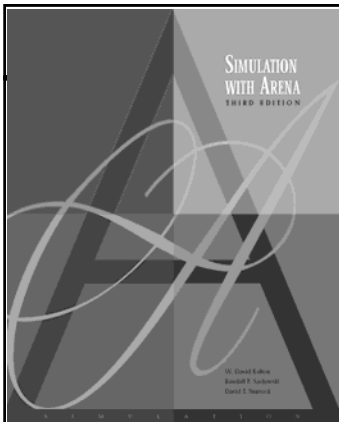


Fundamental Simulation Concepts

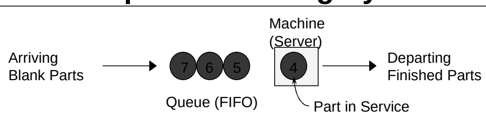
Chapter 2
Subset Only

Last revision June 7, 2003

What We'll Do ...

- **Underlying ideas, methods, and issues in simulation**
- **Software-independent (setting up for Arena)**
- **Centered around an example of a simple processing system**
  - Decompose the problem
  - Terminology
  - Simulation by hand
  - Some basic statistical issues
  - Overview of a simulation study

The System: A Simple Processing System



- **General intent:**
  - Estimate expected production
  - Waiting time in queue, queue length, proportion of time machine is busy
- **Time units**
  - Can use different units in different places … must declare
  - Be careful to check the units when specifying inputs
  - Declare *base time units* for internal calculations, outputs
  - Be reasonable (interpretation, roundoff error)

Model Specifics

- Initially (time 0) empty and idle
- Base time units: minutes
- Input data (assume given for now …), in minutes:

Part Number	Arrival Time	Interarrival Time	Service Time
1	0.00	1.73	2.90
2	1.73	1.35	1.76
3	3.08	0.71	3.39
4	3.79	0.62	4.52
5	4.41	14.28	4.46
6	18.69	0.70	4.36
7	19.39	15.52	2.07
8	34.91	3.15	3.36
9	38.06	1.76	2.37
10	39.82	1.00	5.38
11	40.82	.	.
.	.	.	.
.	.	.	.

- Stop when 20 minutes of (simulated) time have passed

Goals of the Study: Output Performance Measures

- ***Total production*** of parts over the run (***P***)
- ***Average waiting time*** of parts in queue:

$$\frac{\sum_{i=1}^{N} WQ_i}{N}$$

N = no. of parts completing queue wait
WQ_i = waiting time in queue of ith part
Know: $WQ_1 = 0$ (why?)
$N \geq 1$ (why?)

- ***Maximum waiting time*** of parts in queue:

$$\max_{i=1,\ldots,N} WQ_i$$

Goals of the Study: Output Performance Measures (cont'd.)

- ***Time-average number of parts in queue***:

$$\frac{\int_0^{20} Q(t)\, dt}{20}$$

$Q(t)$ = number of parts in queue at time t

- ***Maximum number of parts in queue***: $\max_{0 \leq t \leq 20} Q(t)$
- ***Average*** and ***maximum total time in system*** of parts (a.k.a. ***cycle time***):

$$\frac{\sum_{i=1}^{P} TS_i}{P}, \quad \max_{i=1,\ldots,P} TS_i$$

TS_i = time in system of part i

Goals of the Study: Output Performance Measures (cont'd.)

- ***Utilization* of the machine (proportion of time busy)**

$$\frac{\int_0^{20} B(t)\,dt}{20}, \quad B(t) = \begin{cases} 1 & \text{if the machine is busy at time } t \\ 0 & \text{if the machine is idle at time } t \end{cases}$$

- **Many others possible (information overload?)**

Analysis Options

- **Educated guessing**
  - Average interarrival time = 4.08 minutes
  - Average service time = 3.46 minutes
  - So (on average) parts are being processed faster than they arrive
    - System has a chance of operating in a stable way in the long run, i.e., might not "explode"
    - If all interarrivals and service times were exactly at their mean, there would never be a queue
    - But the data clearly exhibit variability, so a queue could form
  - If we'd had average interarrival < average service time, and this persisted, then queue would explode
  - Truth — between these extremes
  - Guessing has its limits …

Analysis Options (cont'd.)

- **Queueing theory**
  - Requires additional assumptions about the model
  - Popular, simple model: *M/M/1 queue*
    - Interarrival times ~ exponential
    - Service times ~ exponential, indep. of interarrivals
    - Must have E(service) < E(interarrival)
    - Steady-state (long-run, forever)
    - Exact analytic results; e.g., average waiting time in queue is

$$\frac{\mu_S^2}{\mu_A - \mu_S}, \qquad \mu_A = \text{E(interarrival time)}, \quad \mu_S = \text{E(service time)}$$

  - Problems: validity, estimating means, time frame
  - Often useful as first-cut approximation

Mechanistic Simulation

- **Individual operations (arrivals, service times) will occur "exactly" as in reality**
- **Movements, changes occur at the right "time," in the right order**
- **Different pieces interact**
- **Install "observers" to get output performance measures**
- **Concrete, "brute-force" analysis approach**
- **Nothing mysterious or subtle**
  - But a lot of details, bookkeeping
  - Simulation software keeps track of things for you

Arena View of Simulation

- **Later, will talk about "world-views"**
  - The way a modeler views the world
  - The way a coder makes time go by in a simulation
    - A particular implementation approach
- **In the next slides, Arena's view of simulation is described**
  - Not the only way.
  - The authors see simulation through an Arena lens

Pieces of a Simulation Model

- ***Entities***
  - "Players" that move around, change status, affect and are affected by other entities
  - *Dynamic objects* — get created, move around, leave (maybe)
  - Usually represent "real" things
    - Our model: entities are the parts
  - Can have "fake" entities for modeling "tricks"
    - Breakdown demon, break angel
      - Though Arena has built-in ways to model these examples directly
  - Usually have multiple *realizations* floating around
  - Can have different types of entities concurrently
  - Usually, identifying the types of entities is the first thing to do in building a model

Pieces of a Simulation Model (cont'd.)

- ***Attributes***
  - Characteristic of all entities: describe, differentiate
  - All entities have same attribute "slots" but different values for different entities, for example:
    - Time of arrival
    - Due date
    - Priority
    - Color
  - Attribute value tied to a specific entity
  - Like "local" (to entities) variables
  - Some automatic in Arena, some you define

Pieces of a Simulation Model (cont'd.)

- **(Global) *Variables***
  - Reflects a characteristic of the whole model, not of specific entities
  - Used for many different kinds of things
    - Travel time between all station pairs
    - Number of parts in system
    - Simulation clock (built-in Arena variable)
  - Name, value of which there's only one copy for the whole model
  - Not tied to entities
  - Entities can access, change variables
  - Writing on the wall (rewriteable)
  - Some built-in by Arena, you can define others

Pieces of a Simulation Model (cont'd.)

- ***Resources***
  - What entities compete for
    - People
    - Equipment
    - Space
  - Entity *seizes* a resource, uses it, *releases* it
  - Think of a *resource being assigned to an entity*, rather than an entity "belonging to" a resource
  - "A" resource can have several *units* of capacity
    - Seats at a table in a restaurant
    - Identical ticketing agents at an airline counter
  - Number of units of resource can be changed during the simulation

Pieces of a Simulation Model (cont'd.)

- ***Queues***
  - Place for entities to wait when they can't move on (maybe since the resource they want to seize is not available)
  - Have names, often tied to a corresponding resource
  - Can have a finite capacity to model limited space — have to model what to do if an entity shows up to a queue that's already full
  - Usually watch the length of a queue, waiting time in it

Pieces of a Simulation Model (cont'd.)

- ***Statistical accumulators***
  - Variables that "watch" what's happening
  - Depend on output performance measures desired
  - "Passive" in model — don't participate, just watch
  - Many are automatic in Arena, but some you may have to set up and maintain during the simulation
  - At end of simulation, used to compute final output performance measures

Pieces of a Simulation Model (cont'd.)

- **Statistical accumulators for the simple processing system**
  - Number of parts produced so far
  - Total of the waiting times spent in queue so far
  - No. of parts that have gone through the queue
  - Max time in queue we've seen so far
  - Total of times spent in system
  - Max time in system we've seen so far
  - Area so far under queue-length curve $Q(t)$
  - Max of $Q(t)$ so far
  - Area so far under server-busy curve $B(t)$

Simulation Dynamics: The Event-Scheduling "World View"

- **Identify characteristic *events***
- **Decide on *logic* for each type of event to**
  - Effect *state changes* for each event type
  - Observe statistics
  - Update times of future events (maybe of this type, other types)
- **Keep a simulation *clock*, future *event calendar***
- ***Jump* from one event to the next, process, observe statistics, update event calendar**
- **Must specify an appropriate *stopping rule***
- **Usually done with general-purpose programming language (C, FORTRAN, etc.)**

Events for the Simple Processing System

- ***Arrival* of a new part to the system**
  - Update time-persistent statistical accumulators (from last event to now)
    - Area under $Q(t)$
    - Max of $Q(t)$
    - Area under $B(t)$
  - "Mark" arriving part with current time (use later)
  - If machine is idle:
    - Start processing (schedule departure), Make machine busy, Tally waiting time in queue (0)
  - Else (machine is busy):
    - Put part at end of queue, increase queue-length variable
  - Schedule the next arrival event

Events for the Simple Processing System (cont'd.)

- ***Departure* (when a service is completed)**
  - Increment number-produced stat accumulator
  - Compute & tally time in system (now - time of arrival)
  - Update time-persistent statistics (as in arrival event)
  - If queue is non-empty:
    - Take first part out of queue, compute & tally its waiting time in queue, begin service (schedule departure event)
  - Else (queue is empty):
    - Make the machine idle (Note: there will be no departure event scheduled on the future events calendar, which is as desired)

Events for the Simple Processing System (cont'd.)

- ***The End***
  - Update time-persistent statistics (to end of the simulation)
  - Compute final output performance measures using current (= final) values of statistical accumulators
- **After each event, the event calendar's top record is removed to see what time it is, what to do**
- **Also must initialize everything**

Some Additional Specifics for the Simple Processing System

- **Simulation clock variable (internal in Arena)**
- **Event calendar: list of event *records*:**
  - [Entity No., Event Time, Event Type]
  - Keep *ranked* in increasing order on Event Time
  - Next event always in top record
  - Initially, schedule first Arrival, The End (Dep.?)
- **State variables: describe current status**
  - Server status $B(t)$ = 1 for busy, 0 for idle
  - Number of customers in queue $Q(t)$
  - Times of arrival of each customer now in queue (a list of random length)

(Inappropriate snide remark: these guys would have failed in their 1st programming assignment due to poor choice of variable names.)

Simulation by Hand

- **Manually track state variables, statistical accumulators**
- **Use "given" interarrival, service times**
- **Keep track of event calendar**
- **"Lurch" clock from one event to the next**
- **Will omit times in system, "max" computations here (see text for complete details)**

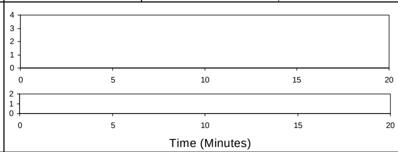
Simulation by Hand: Setup

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)

Q(*t*) graph

B(*t*) graph



Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

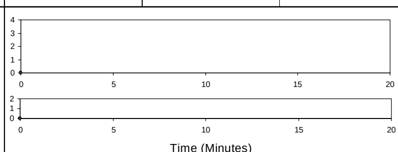
Simulation by Hand: *t* = 0.00, Initialize

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
	0.00	0	0	<empty>	[1, 0.00, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
0	0.00	0.00	0.00

Q(*t*) graph

B(*t*) graph



Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

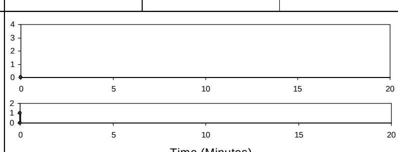
Simulation by Hand: *t* = 0.00, Arrival of Part 1

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
1	0.00	1	0	<empty>	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
1	0.00	0.00	0.00

Q(*t*) graph

B(*t*) graph



Interarrival times	~~1.73~~, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

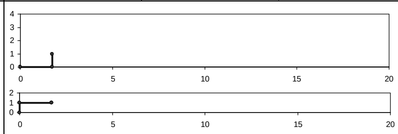
Simulation by Hand: t = 1.73, Arrival of Part 2

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
(2) [1]	1.73	1	1	(1.73)	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
1	0.00	0.00	1.73

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

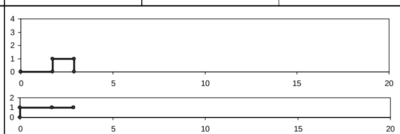
Simulation by Hand: t = 2.90, Departure of Part 1

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
[2]	2.90	1	0	<empty>	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
2	1.17	1.17	2.90

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

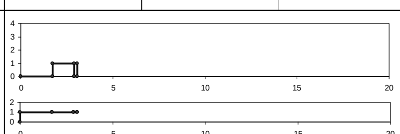
Simulation by Hand: t = 3.08, Arrival of Part 3

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
(3) [2]	3.08	1	1	(3.08)	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
2	1.17	1.17	3.08

$Q(t)$ graph

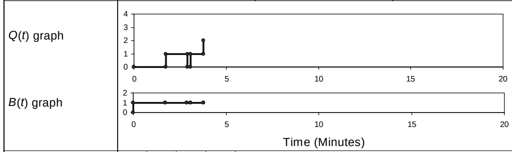
$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: *t* = 3.79, Arrival of Part 4

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
4 3 [2]	3.79	1	2	(3.79, 3.08)	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]
Number of completed waiting times in queue 2	Total of waiting times in queue 1.17		Area under *Q*(*t*) 1.88		Area under *B*(*t*) 3.79



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

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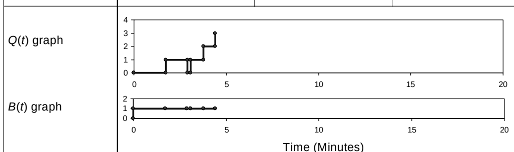
Simulation by Hand: *t* = 4.41, Arrival of Part 5

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
5 4 3 [2]	4.41	1	3	(4.41, 3.79, 3.08)	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]
Number of completed waiting times in queue 2	Total of waiting times in queue 1.17		Area under *Q*(*t*) 3.12		Area under *B*(*t*) 4.41



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

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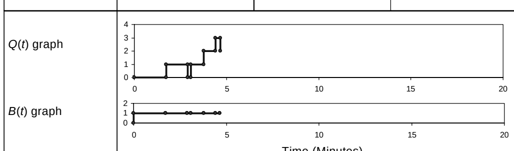
Simulation by Hand: *t* = 4.66, Departure of Part 2

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
5 4 [3]	4.66	1	2	(4.41, 3.79)	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]
Number of completed waiting times in queue 3	Total of waiting times in queue 2.75		Area under *Q*(*t*) 3.87		Area under *B*(*t*) 4.66



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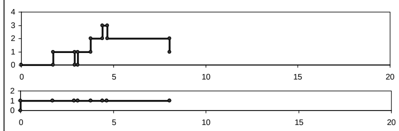
Simulation by Hand: *t* = 8.05, Departure of Part 3

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
5 4	8.05	1	1	(4.41)	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
4	7.01	10.65	8.05

Q(*t*) graph

B(*t*) graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

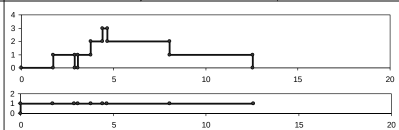
Simulation by Hand: *t* = 12.57, Departure of Part 4

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
5	12.57	1	0	()	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
5	15.17	15.17	12.57

Q(*t*) graph

B(*t*) graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

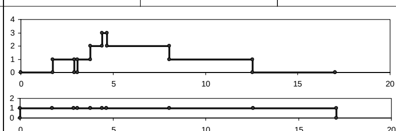
Simulation by Hand: *t* = 17.03, Departure of Part 5

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
	17.03	0	0	()	[6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
5	15.17	15.17	17.03

Q(*t*) graph

B(*t*) graph

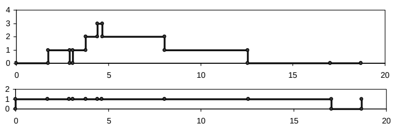


Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: *t* = 18.69, Arrival of Part 6

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
6	18.69	1	0	()	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
6	15.17	15.17	17.03

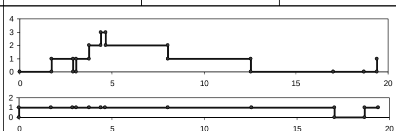


Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: *t* = 19.39, Arrival of Part 7

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
7 6	19.39	1	1	(19.39)	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
6	15.17	15.17	17.73

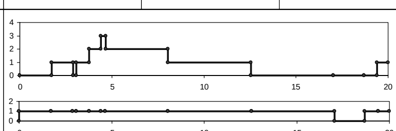


Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: *t* = 20.00, The End

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
7 6	20.00	1	1	(19.39)	[6, 23.05, Dep] [8, 34.91, Arr]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
6	15.17	15.78	18.34



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: Finishing Up

- **Average waiting time in queue:**

$$\frac{\text{Total of times in queue}}{\text{No. of times in queue}} = \frac{15.17}{6} = 2.53 \text{ minutes per part}$$

- **Time-average number in queue:**

$$\frac{\text{Area under } Q(t) \text{ curve}}{\text{Final clock value}} = \frac{15.78}{20} = 0.79 \text{ part}$$

- **Utilization of drill press:**

$$\frac{\text{Area under } B(t) \text{ curve}}{\text{Final clock value}} = \frac{18.34}{20} = 0.92 \text{ (dimensionless)}$$

Complete Record of the Hand Simulation

Just-Finished Event			Variables		Attributes: Arrival Times		Statistical Accumulators									Event Calendar
Entity No.	Time t	Event Type	Q(t)	B(t)	(In Queue)	In Service	P	N	ΣWQ	WQ*	ΣTS	TS*	∫Q	Q*	∫B	[Entity No., Time, Type]
–	0.00	Init	0	0	( )	–	0	0	0.00	0.00	0.00	0.00	0.00	0	0.00	[1, 0.00, Arr] [–, 20.00, End]
1	0.00	Arr	0	1	( )	0.00	0	1	0.00	0.00	0.00	0.00	0.00	0	0.00	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]
2	1.73	Arr	1	1	(1.73)	0.00	0	1	0.00	0.00	0.00	0.00	0.00	1	1.73	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00, End]
1	2.90	Dep	0	1	( )	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	2.90	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]
3	3.08	Arr	1	1	(3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	3.08	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]
4	3.79	Arr	2	1	(3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.88	2	3.79	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]
5	4.41	Arr	3	1	(4.41, 3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	3.12	3	4.41	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]
2	4.66	Dep	2	1	(4.41, 3.79)	3.08	2	3	2.75	1.58	5.83	2.93	3.87	3	4.66	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]
3	8.05	Dep	1	1	(4.41)	3.79	3	4	7.01	4.26	10.80	4.97	10.65	3	8.05	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]
4	12.57	Dep	0	1	( )	4.41	4	5	15.17	8.16	19.58	8.78	15.17	3	12.57	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]
5	17.03	Dep	0	0	( )	–	5	5	15.17	8.16	32.20	12.62	15.17	3	17.03	[6, 18.69, Arr] [–, 20.00, End]
6	18.69	Arr	0	1	( )	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.03	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]
7	19.39	Arr	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.73	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]
–	20.00	End	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.78	3	18.34	[6, 23.05, Dep] [8, 34.91, Arr]

Event-Scheduling Logic via Programming

- **Clearly well suited to standard programming language**
- **Often use "utility" libraries for:**
  - List processing
  - Random-number generation
  - Random-variate generation
  - Statistics collection
  - Event-list and clock management
  - Summary and output
- **Main program ties it together, executes events in order**

Simulation Dynamics: The Process-Interaction World View

- **Identify characteristic *entities* in the system**
- **Multiple copies of entities co-exist, interact, compete**
- **"Code" is non-procedural**
- **Tell a "story" about what happens to a "typical" entity**
- **May have many types of entities, "fake" entities for things like machine breakdowns**
- **Usually requires special simulation software**
  - Underneath, still executed as event-scheduling
- **The view normally taken by Arena**
  - Arena translates your model description into a program in the SIMAN simulation language for execution

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Randomness in Simulation

- **The above was just one "replication" — a sample of size one (not worth much)**
- **Made a total of five replications:**

Performance Measure	Replication 1	2	3	4	5	Sample Avg.	Std. Dev.	95% Half Width
Total production	5	3	6	2	3	3.80	1.64	2.04
Average waiting time in queue	2.53	1.19	1.03	1.62	0.00	1.27	0.92	1.14
Maximum waiting time in queue	8.16	3.56	2.97	3.24	0.00	3.59*	2.93*	3.63*
Average total time in system	6.44	5.10	4.16	6.71	4.26	5.33	1.19	1.48
Maximum total time in system	12.62	6.63	6.27	7.71	4.96	7.64*	2.95*	3.67*
Time-average number of parts in queue	0.79	0.18	0.36	0.16	0.05	0.31	0.29	0.36
Maximum number of parts in queue	3	1	2	1	1	1.60*	0.89*	1.11*
Drill-press utilization	0.92	0.59	0.90	0.51	0.70	0.72	0.18	0.23

Note substantial variability across replications

- **Confidence intervals for expected values:**
  - In general, $\bar{X} \pm t_{n-1,1-\alpha/2} s / \sqrt{n}$
  - For expected total production, $3.80 \pm (2.776)(1.64 / \sqrt{5})$
    3.80 ± 2.04

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Comparing Alternatives

- **Usually, simulation is used for more than just a single model "configuration"**
- **Often want to compare alternatives, select or search for the best (via some criterion)**
- **Simple processing system: What would happen if the arrival rate were to double?**
  - Cut interarrival times in half
  - Rerun the model for double-time arrivals
  - Make five replications

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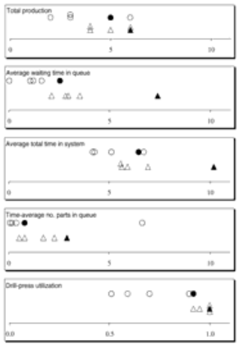
Results: Original vs. Double-Time Arrivals



- Original – circles
- Double-time – triangles
- Replication 1 – filled in
- Replications 2-5 – hollow
- Note variability
- Danger of making decisions based on one (first) replication
- Hard to see if there are really differences
- Need: Statistical analysis of simulation output data

Overview of a Simulation Study

- Understand the system
- Be clear about the goals
- Formulate the model representation
- Translate into modeling software
- Verify "program"
- Validate model
- Design experiments
- Make runs
- Analyze, get insight, document results