CS 475/575 Slide Set 7

Random Variates 2
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Uniform distribution

pdf $f(x) = \begin{cases} 1/(b-a) & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

cdf $F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{1}{b-a}dx = \frac{1}{b-a}\int_{-\infty}^{x} dx = \frac{x}{b-a} + C$

When X=a, F(x)=0, and C=(-a)/(b-a), so

F(x) = (x-a)/(b-a)

so r=(x-a)/(b-a) or x=r(b-a)+a where r is U(0,1)

$\mu = \int_{-\infty}^{\infty} xf(x)dx = \frac{a+b}{2}, \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx, \sigma = \frac{b-a}{\sqrt{12}}$

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Exponential distribution - 1

Assumptions:

1. The probability that an event occurs during $[t, t+\Delta t]$ is $\alpha\Delta t$
2. α is constant and independent of t.
3. The probability that more than one event occurs in $[t, t+\Delta t]$ approaches 0 as $\Delta t \rightarrow 0$

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Exponential dist. - 2

If X is the length of time between successive events then X is said to have an *exponential distribution* and its PDF is

$$f(x) = \alpha e^{-\alpha x},\ (\alpha > 0, x > 0)$$

The CDF is

$$F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \alpha e^{-\alpha x}dx = 1 - e^{-\alpha x}$$

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more exp.

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x\alpha e^{-\alpha x}dx = \frac{1}{\alpha}$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \frac{1}{\alpha})^2 \alpha e^{-\alpha x}dx = \frac{1}{\alpha^2} = \mu^2$$

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even more exp.

Note that since F is expressible in closed form (that is, $F(x) = 1 - e^{-\alpha x}$), we can find $F^{-1}(x)$:

$$y = 1 - e^{-\alpha x}$$

$$e^{-\alpha x} = 1 - y$$

$$-\alpha x \ln e = \ln(1 - y)$$

$$x = -\frac{1}{\alpha}\ln(1 - y)$$

Since (1-r) is identically distributed to r, we use

$$x = -\frac{1}{\alpha}\ln r,\ \text{for}\ r \approx U(0,1)$$

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Normal

Recall:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

with mean μ, variance σ^2, std. dev. σ

Cannot use inverse CDF technique since we have not closed form representation, but do have at least 2 techniques.

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Normal - technique 1

Recall *central limit theorem:* the probability distribution of the sum of N iid random variates with mean μ and variance σ^2 tends toward a normal distribution with mean Nμ and variance σ^2/n.

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normal - tech 1 cont.

Thus if $r_i \approx U(0,1)$ then

$$x = \frac{\sum_{i=1}^{k} r_i - k\mu}{\sqrt{k\sigma^2}} \text{ is } N(0,1)$$

Since for U(a,b), μ =(a+b)/2, $\sigma = (b-a)/\sqrt{12}$,

$$x = \frac{\sum_{i=1}^{k} r_i - \frac{k}{2}}{\sqrt{k/12}}$$ so pick k=12 to get $x = \sum_{i=1}^{12} r_i - 6$

Problem: not good beyond $\pm 3\sigma$

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Normal - technique 2

Requires generation of normals in pairs:

1. Generate two U(0,1) random variates, μ_1, μ_2
2. Let $x_1 = \sqrt{-2 \ln \mu_1} \cos 2\pi\mu_2$
3. Let $x_2 = \sqrt{-2 \ln \mu_1} \sin 2\pi\mu_2$

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using Arena to fit distributions

⌘see text, pp. 156-163

⌘steps:

- open Arena
- from Tools, select Input Analyzer
- select “File,” “new”, then “use existing” icon.
- select fit, then “fit all” or a particular distribution

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