CS381 Introduction to Discrete Structures

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Assignment 3

Due: March 17, 2010, no later than 3:00pm

Objective: The main objective of your third assignment is for you to practice reasoning about functions and their various properties. In dealing with functions, you will find it useful to bring to bear set-theoretical concepts that you have mastered.

Let me remind you that the assignment is strictly personal and giving/receiving undue help is a violation of the Honor Code of Old Dominion University and will be dealt with accordingly. If you are not sure what constitutes a violation of the Honor Code please do not hesitate to ask.

Statement of your assignment:

Problem 1. [20%] Let $A$ and $B$ be sets and let $f : A \rightarrow B$ be a function from $A$ to $B$. Let $S$ and $T$ be arbitrary subsets of $A$. Find a counter-example to the claim that

$$f(S \cap T) = f(S) \cap f(T).$$

Problem 2. [20%] Determine which of the following functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$

2.1. $f(x) = -x + 1$;
2.2. $f(x) = x^2 - 2x + 1$.

Problem 3. [20%] Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and let $f(x) > 0$. Show that $f(x)$ is strictly increasing if and only if $\frac{1}{f(x)}$ is strictly decreasing.

Problem 4. [20%] Let $f$ and $g$ be functions from $\mathbb{R}$ to $\mathbb{R}$ such that $f(x) = ax + b$ and $g(x) = cx + d$ where $a, b, c, d$ are constants. What relation should exist between $a, b, c, d$ for $f \circ g = g \circ f$ to hold?

Problem 5. [20%] Give examples of functions from $\mathbb{N}$ to $\mathbb{N}$ that are

5.1. One-to-one but not onto;
5.2. Onto but not one-to-one;
5.3. Both one-to-one not onto;
5.4. Neither one-to-one not onto.