Using Perturbed QR Factorizations to Solve Linear Least-Squares Problems

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February 2007

Abstract

We propose and analyze a new tool to help solve sparse linear least-squares problems $\min_x \|Ax-b\|_2$. Our method is based on a sparse QR factorization of a low-rank perturbation $\ddot{A}$ of A. More precisely, we show that the$R$ factor of $\ddot{A}$ is an effective preconditioner for the least-squares problem $\min_x \|Ax-b\|_2$, when solved using LSQR. We purpose applications for the new technique. When A is rank deficient we can add rows to ensure that the preconditioner is well-conditioned without column pivoting. When A is sparse except for a few dense rows we can drop the dense rows in $\ddot{A}$. Another application is solving an updated or downdated problem. If R is a good preconditioner for the original problem A, it is a good preconditioner for the updated/downdated problem $\ddot{A}$. We can also solve what-if scenarios, where we want to find the solution if a column of the original matrix is changed/removed. We present a spectral theory that analyzes the generalized spectrum of the pencil (A^*A, R^*R) and analyze the applications.

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