

Solutions

CS 381 Midterm Review

1. Prove that $((a \rightarrow \neg b) \wedge (a \wedge b))$ is a contradiction using logical equivalences.
2 pts.

Proof 1:

$$\textcircled{1} (\neg a \vee \neg b) \wedge (a \wedge b)$$

Definition of implication

$$\textcircled{2} \neg(a \wedge b) \wedge (a \wedge b)$$

De Morgan's

$$\textcircled{3} F$$

Trivial contradiction

Proof 2:

$$\textcircled{1} (\neg a \vee \neg b) \wedge (a \wedge b)$$

Definition of implication

$$\textcircled{2} ((\neg a \vee \neg b) \wedge a) \wedge b$$

Associative law

$$\textcircled{3} (a \wedge (\neg a \vee \neg b)) \wedge b$$

Commutative law

$$\textcircled{4} ((a \wedge \neg a) \vee (a \wedge \neg b)) \wedge b$$

Distributive law

$$\textcircled{5} (F \vee (a \wedge \neg b)) \wedge b$$

Trivial contradiction

$$\textcircled{6} (a \wedge \neg b) \wedge b$$

Identity

~~$$\textcircled{7} (a \wedge \neg b) \wedge b$$~~

~~Commutative law~~

~~$$\textcircled{8} (a \wedge \neg b) \wedge b$$~~

$$\textcircled{7} a \wedge (\neg b \wedge b)$$

Associative law

$$\textcircled{8} a \wedge F$$

Trivial contradiction

$$\textcircled{9} F$$

Domination