

Equivalence Rules

9. Specify and prove big-O, big-Omega (Ω), and big-Theta (Θ) bounds for the following function:

4 pts. $h(x) = 5x^2 + 6x$

$h(x)$ is $O(x^2)$, $\Omega(x^2)$, and $\Theta(x^2)$.

Proof of $O(x^2)$ upper bound:

Show that $h(x) \leq Cx^2$ for $x > k$, where $C=6$ and $k=6$.

$$5x^2 + 6x \leq 6x^2$$

$$6x \leq x^2$$

$$6 \leq x \quad \text{True for } x > 6.$$

Proof of $\Omega(x^2)$ lower bound:

Show that $h(x) \geq Cx^2$ for $x > k$, where $C=4$ and $k=1$.

$$5x^2 + 6x \geq 4x^2$$

$$x^2 + 6x \geq 0 \quad \text{True for } x > 1.$$

$h(x)$ must be $\Theta(x^2)$ (of exact order x^2)

since $h(x)$ has an upper bound $O(x^2)$

and a lower bound $\Omega(x^2)$.