

⑥ Provide a direct proof.

$$\text{Premise 1: } (\neg(g \wedge h)) \wedge (\neg(g \vee h))$$

$$\text{Premise 2: } (h \rightarrow \neg j)$$

$$\text{Premise 3: } (k \vee j)$$

$$\text{Prove: } \neg g \rightarrow k$$

$$1) (\neg g \vee \neg h) \wedge (\neg g \wedge \neg h)$$

De Morgan's applied to Premise 1.

$$2) (\neg g \wedge \neg h)$$

Simplification applied to (1)

$$3) \neg h$$

Simplification applied to (2)

$$4) \neg j$$

Modus ponens applied to (3) and Premise 2

$$5) k$$

Disjunctive syllogism applied to (4) and Premise 3

In this case I was able to prove that ~~given~~ given the premises, k is always true.

I did not have to assume the antecedent of the implication was true ($\neg g$), though this would be allowed.