

1 pt.

7. x is a variable whose universe of discourse contains objects $\{a, b, c, d\}$.
Is the following a valid proof? If not, explain why.

Premise 1: $\forall x (B(x) \rightarrow C(x))$

Premise 2: $B(a) \rightarrow D(a)$

Premise 3: $B(a)$

Theorem: $\forall x D(x) \rightarrow C(x)$

Proof:

1. $B(a) \rightarrow C(a)$ Universal instantiation applies to Premise 1.
2. $D(a) \rightarrow C(a)$ Modus ponens applied to Proof line 1 and Premise 2.
3. $\forall x D(x) \rightarrow C(x)$ Universal generalization applied to Proof line 2.

This is an incorrect proof.

1. Modus ponens does not allow the conclusion of $D(a) \rightarrow C(a)$ on line 2.
2. Universal generalization cannot be applied as in line 3.

- 1 pt. Bonus: Describe a universe of discourse for variables x and y for which the following predicate is true:

$$Q(x,y) \text{ means } \frac{x}{y} < xy$$

One possibility:

$$(x \in \mathbb{R}) \wedge (x > 0)$$

$$(y \in \mathbb{R}) \wedge (y > 1)$$