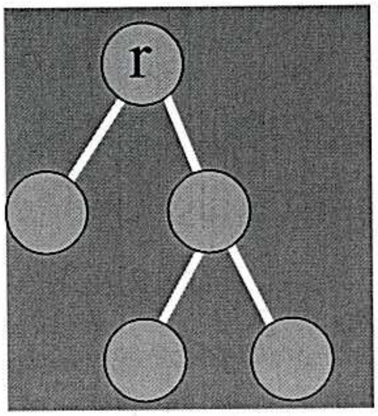
The definition of a full binary tree is:

Basis step:

The simplest full binary tree is a single node, r.

Recursive step:

If T_1 and T_2 are full binary trees, another full binary tree can be constructed from T_1, T_2, and a new root node r by adding an edge from r to the root of T_1 and adding an edge from r to the root of T_2.



Example:
The binary tree shown on the left has root node r.
It has 5 nodes and a height of 2.

4. Use structural induction and the definition of a full binary tree given above to prove that $n(T) \geq 2h(T)+1$, where T is a full binary tree, n(T) equals the number of nodes in T, and h(T) equals the height of T.

3 pts.

Basis step: T = one node

$h(T) = 0$

$n(T) = 1$

$1 \geq 2(0) + 1 = 1$

$\therefore n(T) \geq 2h(T) + 1$

Inductive (recursive) step:

Assume T_1 and T_2 are full binary trees, and that $n(T_1) \geq 2h(T_1)+1$ and $n(T_2) \geq 2h(T_2)+1$

Prove that

$n(T) \geq 2h(T)+1$ where T is a new full binary tree formed by a new root node r, with edges to T_1 and T_2.