Sec. 3.4

#6b) $f(0)=1$

$f(1)=0$

$f(2)=2$

$f(n)=2f(n-3)$ for $n\geq 3$ (Recursive def. of function)

Thus, the sequence of numbers produced by this function is:

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(n)$ | 1 | 0 | 2 | 2 | 0 | 4 | 4 | 0 | 8 | ... |

A non-recursive definition for this function is:

$$f(n)=\begin{cases} 2^{n/3} & \text{if } n \bmod 3=0 \\ 0 & \text{if } n \bmod 3=1 \\ 2^{(n+1)/3} & \text{if } n \bmod 3=2 \end{cases}$$

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Inductive proof that the recursive and non-recursive definitions of $f(n)$ are equivalent:

Basis cases: $n=0$ $\quad 2^{0/3}=2^0=1=f(0)$

$n=1$ $\quad 0=f(1)$

$n=2$ $\quad 2^{(2+1)/3}=2^{3/3}=2^1=2=f(2)$

Inductive step:

Assume the recursive and non-recursive formulas are equivalent for $0\leq n\leq k$, where $k\geq 2$.

Prove the recursive and non-recursive formulas are equivalent for $n=k+1$.

There are three possibilities:

A) $(k+1) \bmod 3=0$

B) $(k+1) \bmod 3=1$

C) $(k+1) \bmod 3=2$