

Midterm Presentation - Problem L

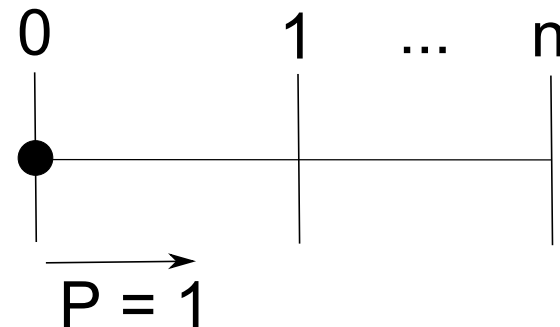
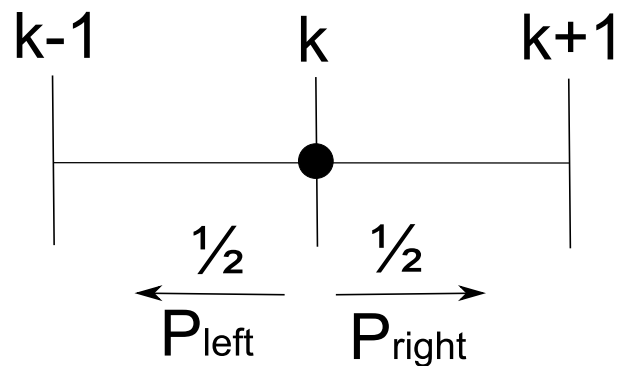
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CS895 - Stochastic Modeling

Old Dominion University

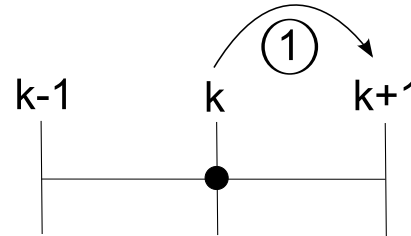
Problem L: Consider a particle that moves along the set $0, 1, \dots, n$ of integers so that at each step it is equally likely to move to any of its neighbors. Starting at 0, show that the expected number of steps it takes to reach n is n^2 .



I_1 = Number of iterations required for particle to go from 0 to 1

I_k = Number of iterations required for particle to go from $k - 1$ to k

$$E[I_1] = 1$$



When $k > 1$, use Law of Total Expectation:

$$E[I_k] = \sum P(K = k) \times E(\text{At } I_k | K = k)$$

$$= P(\text{At } I_k | \text{At } I_{k-1}) \times E[\text{Directly to } I_k | I_{k-1}] + P(\text{At } I_k | \text{At } I_{k-1}) \times E[\text{Indirectly to } I_k | I_{k-1}]$$

$$= \frac{1}{2} \times 1 + \frac{1}{2} E[1 + I_{k-1} + I_k]$$

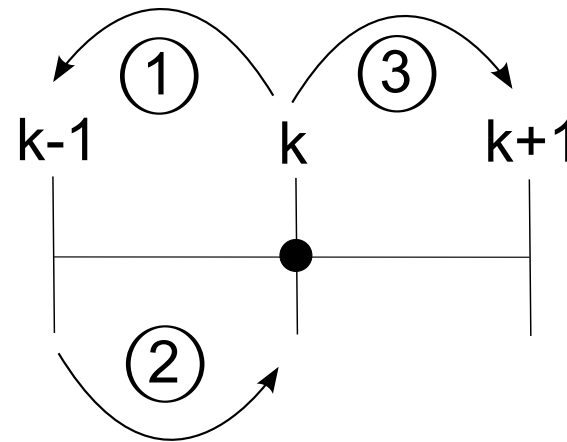
$$= \frac{1}{2} + \frac{1}{2} E[1] + \frac{1}{2} E[I_{k-1}] + \frac{1}{2} E[I_k]$$

$$= \frac{1}{2} + \frac{1}{2} \times 1 + \frac{1}{2} E[I_{k-1}] + \frac{1}{2} E[I_k] \rightarrow$$

$$\frac{1}{2} E[I_k] = 1 + \frac{1}{2} E[I_{k-1}] \rightarrow$$

$$E[I_k] = 2 + E[I_{k-1}]$$

$$E[I_k] - E[I_{k-1}] = 2$$



So:

$$\left. \begin{array}{l} E[I_2] - E[I_1] = 2 \\ E[I_3] - E[I_2] = 2 \\ E[I_4] - E[I_3] = 2 \\ \dots \\ E[I_k] - E[I_{k-1}] = 2 \end{array} \right\} k - 1 \text{Expectations}$$

(1)

To go to I_k from I_1 : $E[I_k] - E[I_1] = 2(k - 1)$, All integers between 1 and k must be visited to get from 1 to k .

$$E[I_k] = 2(k - 1) + E[I_1] = 2k - 2 + 1 = 2k - 1 \quad (2)$$

$$E\left[\sum_{k=1}^n I_k\right] = \sum_{k=1}^n E[I_k] = \sum_{k=1}^n (2k - 1) = \quad (3)$$

$$\frac{2n^2 + 2n}{2} - n \quad (4)$$

$$= \frac{2n(n + 1)}{2} - n \quad (5)$$

$$= n(n + 1) - n \quad (6)$$

$$= n^2 + n - n \quad (7)$$

$$= n^2 \quad (8)$$