NASA Technical Memorandum 4701

Buckling Analysis of Debonded Sandwich Panel Under Compression

David W. Sleight and John T. Wang
Langley Research Center • Hampton, Virginia

National Aeronautics and Space Administration
Langley Research Center • Hampton, Virginia 23681-0001

December 1995

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Abstract

A sandwich panel with initial through-the-width debonds is analyzed to study the buckling of its faceskin when subject to an in-plane compressive load. The debonded faceskin is modeled as a beam on a Winkler elastic foundation in which the springs of the elastic foundation represent the sandwich foam. The Rayleigh-Ritz and finite-difference methods are used to predict the critical buckling load for various debond lengths and stiffnesses of the sandwich foam. The accuracy of the methods is assessed with a plane-strain finite-element analysis. Results indicate that the elastic foundation approach underpredicts buckling loads for sandwich panels with isotropic foam cores.

Introduction

Sandwich construction has been used in aircraft applications over the last 40 years because of its light weight, high bending rigidity, and good fatigue properties (ref. 1). However, debonding of the faceskin from the core, which can be by impact damage or defects in the manufacturing process, significantly degrades sandwich construction performance. In particular, debonding can significantly affect the compression stability and strength of the sandwich structure. Moreover, the presence of debonding is a major concern in sandwich structures because the debonded region may grow under compression (ref. 2).

There have been several studies on the buckling of debonded sandwich and delaminated composite structures loaded under compression (refs. 2–8). Webster investigated the buckling loads of composite laminates with circular debond defects using the Rayleigh-Ritz method (ref. 3). Chai et al. (refs. 4 and 5) studied buckling and postbuckling of delaminated composite plates by modeling the failure in one and two dimensions. Hwu and Hu (ref. 2) and Somers et al. (ref. 6) derived closed-form solutions to study the buckling and postbuckling of delaminated sandwich beams. They applied elastic fracture mechanics (LEFM), using J-integral and Griffith criterion to compute strain energy release rates for predicting debond growth. Vizzini and Lagace (ref. 7) studied sublaminate delamination in composite laminates. In their approach, a one-dimensional (1-D) model of a delaminated sublaminate was developed in which the sublaminate rested on an elastic foundation that modeled the resin layer. The buckling load of the sublaminate was predicted using the Rayleigh-Ritz energy method. Kim and Dharan (ref. 8) used the approach developed by Vizzini and Lagace with LEFM to study faceskin debonding for composite sandwich panels.

This paper examines a sandwich panel with initial through-the-width debonds between the faceskin and sandwich foam core. (See fig. 1.) The buckling load of the panel is computed from a 1-D model, which models a sandwich panel strip as a beam (faceskin) on an elastic foundation (core material), and a two-dimensional (2-D) model, which models the cross section of the panel. The Rayleigh-Ritz and finite-difference methods are used to analyze the 1-D model, while the finite-element method using 4-node quadrilateral plane-strain elements is used to analyze the 2-D model.

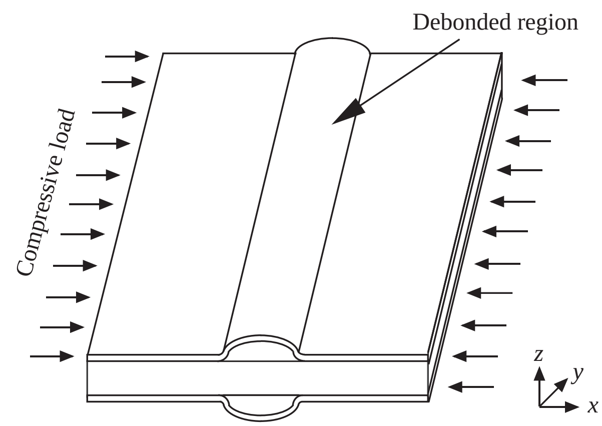


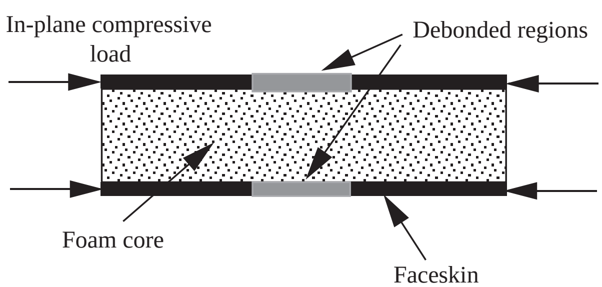
Figure 1. Debonded sandwich panel.

Although many investigators used the 1-D beam-on-elastic-foundation model to study the sandwich panel debond problem, little research has been performed (to the authors' knowledge) to assess the adequacy of modeling the foam core material as elastic springs by more accurate analysis methods, such as finite-element analysis. In the finite-element model, the foam core material is modeled as an isotropic material. Therefore, because the finite-element model properly includes the behavior of the faceskin and foam core, more accurate predictions of buckling will result.

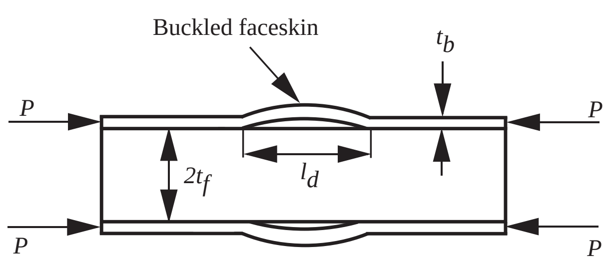
This paper presents three different numerical methods (Rayleigh-Ritz, finite-difference, and finite-element) for determining the buckling load of a sandwich panel with initial through-the-width debonds. Parametric studies investigating the effects of sandwich foam stiffness and debond length on the buckling load of the debonded faceskin are performed. Finally, the assumption of using the elastic foundation approach to model the foam core is evaluated by comparing the Rayleigh-Ritz and finite-difference results from the 1-D elastic beam foundation model to the results from the 2-D plane-strain finite-element model.

Numerical Methods for Determining Buckling Load

As previously mentioned, the Rayleigh-Ritz, finite-difference, and finite-element methods were used to analyze the buckling of the debonded sandwich panel. In the Rayleigh-Ritz and finite-difference methods, a sandwich panel strip was modeled as a beam on an elastic foundation. In the finite-element method, 2-D plane-strain quadrilateral elements were used to model the cross section of the debonded sandwich panel.



(a) Sandwich beam with initial symmetric debonded regions between core and faceskin.



(b) Buckled shape of sandwich beam.

Figure 2. Buckling of debonded sandwich panel strip modeled as a beam on an elastic foundation.

Beam-on-Elastic-Foundation Model

Figure 2(a) shows a strip of unit width b of the sandwich panel loaded in compression with symmetric through-the-width debonds between the faceskins and core. The debonded region of length l_d is located in the middle of the sandwich panel, and the applied compressive load is P, as shown in figure 2(b). The thickness of the faceskins is denoted by t_b, and the thickness of the sandwich core is denoted by $2t_f$. Due to symmetry, only the upper half of the sandwich panel strip of length L is modeled, as shown in figure 3. The Young's modulus and the moment of inertia of the faceskins are denoted by E_b and $I = t_b^3/12$, respectively. The bonded regions l_f have foundation support. The faceskin, modeled as a beam, is partially supported by an elastic foundation with modulus k and has simple supported boundary conditions at its ends.

Rayleigh-Ritz method. The total potential energy of the beam-on-elastic-foundation system is expressed as

$$\Pi = U_m + U_b + U_f + \Omega \tag{1}$$

where U_m and U_b are membrane and bending energies, respectively, U_f is the potential energy of the elastic foundation, and Ω is the potential energy of the compressive load P. These energy terms are defined as

$$U_m = \frac{E_b A}{2}\int_0^L \varepsilon_b^{\,2}\,dx \tag{2a}$$

$$U_b = \frac{E_b I}{2}\int_0^L (\bar{w}'')^2\,dx \tag{2b}$$

$$U_f = \frac{1}{2}\int_0^L k(x)\bar{w}^2\,dx \tag{2c}$$

$$\Omega = P\int_0^L \bar{u}'\,dx \tag{2d}$$

where A is the cross-sectional area of the faceskin and $\bar{u}$, and $\bar{w}$ are the longitudinal and out-of-plane deflections, respectively (ref. 9). The extensional strain of the faceskin can be written as

$$\varepsilon_b = \bar{u}' + \frac{1}{2}(w')^2 \tag{3}$$

Introduction into equation (1) gives

$$\Pi = \int_0^L \left[\frac{E_b A}{2}\left(\bar{u}' + \frac{1}{2}\bar{w}'^2\right)^2 + \frac{E_b I}{2}(\bar{w}'')^2 + \frac{1}{2}k\bar{w}^2 + P\bar{u}'\right]dx \tag{4}$$

The elastic foundation modulus $k(x)$ in equation (2c) is given by

$$k(x) = K\psi(x) \tag{5}$$

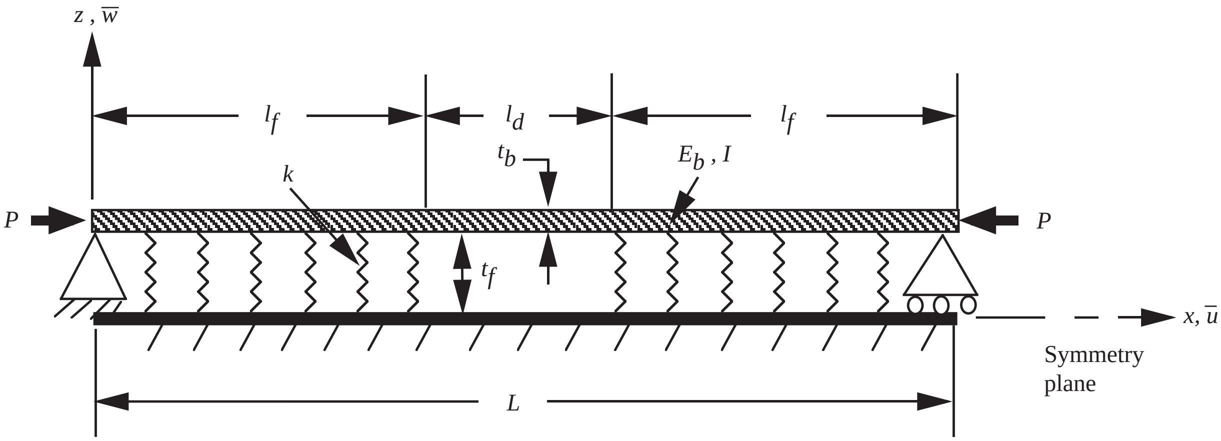


Figure 3. Elastic foundation model of debonded sandwich beam.

where K is a constant and can be represented as $E_f\, b/t_f$ and E_f is the Young's modulus of the sandwich foam core. The variable $\psi(x)$ is defined as

$$\psi(x) = \begin{cases} 1 \text{ if } 0 \le x \le l_f \\ 0 \text{ if } l_f \le x \le L - l_f \\ 1 \text{ if } L - l_f \le x \le L \end{cases} \tag{6}$$

For stability of a system in equilibrium, the second variation of potential energy needs to be positive semidefinite,

$$\delta^2 \Pi[\bar{w}] \ge 0 \tag{7}$$

When the second variation of potential energy is zero, a neutral equilibrium configuration exists and indicates the onset of buckling. This point is characterized as the bifurcation point (ref. 10). The Trefftz criterion states that equation (7) is identically satisfied when the variation of the second variation is zero (ref. 11).

The expression for $\delta^2\Pi$ is obtained by perturbing the system in equation (4) such that

$$\left.\begin{aligned} \bar{u} &\to u_0 + u \\ \bar{w} &\to w_0 + w \end{aligned}\right\} \tag{8}$$

where u_0 and w_0 represent the prebuckling configuration and the variations $u(x)$ and $w(x)$ are infinitesimally small increments. For the prebuckling configuration,

$$\left.\begin{aligned} u_0 &= -\frac{P}{E_b A} x \\ w_0 &= 0 \end{aligned}\right\} \tag{9}$$

Introducing equations (8) and (9) into equation (4), assuming inextensional buckling (i.e., $u = 0$), and collecting the second-order terms gives the second variation of the potential energy:

$$\delta^2 \Pi = \int_0^L [E_b I (w'')^2 + K \psi w^2 - P (w')^2] dx \tag{10}$$

In the Rayleigh-Ritz method, the out-of-plane deformation at the onset of buckling w of the panel faceskin is approximated as

$$w(x) = \sum_{m=1}^{M} A_m \phi_m(x) \tag{11}$$

where ϕ_m are selected functions that satisfy the geometric boundary conditions and A_m represents their amplitudes. For simple support boundary conditions, a sinusoidal function is used:

$$\phi_m(x) = \sin \frac{m\pi x}{L} \tag{12}$$

The Trefftz criterion used with the Rayleigh-Ritz method to determine the buckling load can be written as

$$\frac{\partial \delta^2 \Pi[w]}{\partial A_m} = 0 \qquad (m = 1, 2, \ldots, M) \tag{13}$$

Dimensional parameters are nondimensionalized by the length L as follows:

$$\left.\begin{aligned} \xi &= x/L \\ \eta &= l_f/L \\ \mu &= l_d/L = 1 - 2\eta \end{aligned}\right\} \tag{14}$$

The variable η is the foundation ratio, where a value of zero is equivalent to a faceskin (beam) with no foundation (unsupported) and a value of one-half is equivalent to a fully supported faceskin (beam). The variable μ is the debond ratio and represents the fractional amount of the strip which is debonded. The functions ϕ_m in equation (12) and ψ in equation (6) become $\tilde{\phi}_m$ and $\tilde{\psi}$, respectively, after the nondimensionalized parameters in equation (7) are used. Incorporating equation (13) gives the eigenvalue problem:

$$\left\{[\tilde{U}_b] + \frac{F}{\mu^4}[\tilde{U}_f]\right\}^{-1}[\tilde{\Omega}]\{A\} = \lambda\{A\} \qquad (15)$$

where the matrix elements are

$$\tilde{U}_{b_{ij}} = \int_0^1 \frac{d^2\tilde{\phi}_i}{d\xi^2}\frac{d^2\tilde{\phi}_j}{d\xi^2}\,d\xi \qquad (16a)$$

$$\tilde{U}_{f_{ij}} = \int_0^1 \tilde{\psi}\tilde{\phi}_i\tilde{\phi}_j\,d\xi \qquad (16b)$$

$$\tilde{\Omega}_{ij} = \int_0^1 \frac{d\tilde{\phi}_i}{d\xi}\frac{d\tilde{\phi}_j}{d\xi}\,d\xi \qquad (16c)$$

and

$$\lambda = \frac{1}{\tilde{P}} = \frac{E_bI}{PL^2} \qquad (17a)$$

$$F = \frac{Kl_d^4}{E_bI} \qquad (17b)$$

The terms $\tilde{P}$ and F are, respectively, the dimensionless load and foundation stiffness parameters. Solving equation (15) for the largest eigenvalue λ yields the lowest buckling load P_{cr}.

Finite-difference method. An alternative method for examining the elastic stability of a sandwich debond is the finite-difference method (ref. 10). The governing equations at the onset of buckling are

$$\left.\begin{aligned} &w^{IV} + \frac{P}{EI}w'' = 0 && \text{(for debonded region)} \\ &w^{IV} + \frac{P}{EI}w'' + \frac{k}{EI}w = 0 && \text{(for bonded region)} \end{aligned}\right\} \qquad (18)$$

In this method, $N + 4$ discrete evenly spaced gridpoints x_{-1} to x_{N+2} are used with spacing Δ as shown in figure 4. Derivatives of w are approximated by

$$\left.\begin{aligned} w''_n &= \frac{w_{n+1} - 2w_n + w_{n-1}}{\Delta^2} \\ w_n^{IV} &= \frac{w_{n+2} - 4w_{n+1} + 6w_n - 4w_{n-1} + w_{n-2}}{\Delta^4} \end{aligned}\right\} \qquad (19)$$

with simply supported boundary conditions

$$\left.\begin{aligned} w_0 &= 0 \\ w_{-1} &= -w_1 \\ w_{N+1} &= 0 \\ w_N &= -w_{N+2} \end{aligned}\right\} \qquad (20)$$

Substituting equation (19) into equation (18) for points x_1 to x_N along with the boundary conditions in equation (20) provides $N + 4$ simultaneous algebraic equations for w_{-1} to w_{N+2}. The nontrivial solution for the buckling load is represented by the lowest value of P for which the determinant of the coefficients of the N algebraic equations vanishes (ref. 9).

Finite-Element Model

Plane-strain finite-element analysis. Due to symmetry about the panel's half-length ($x = L/2$) and thickness, only a quarter of the debonded sandwich panel cross section was modeled by 4-noded quadrilateral assumed natural-coordinate strain (ANS) finite elements, as shown in figure 5 (refs. 12 and 13). The element is

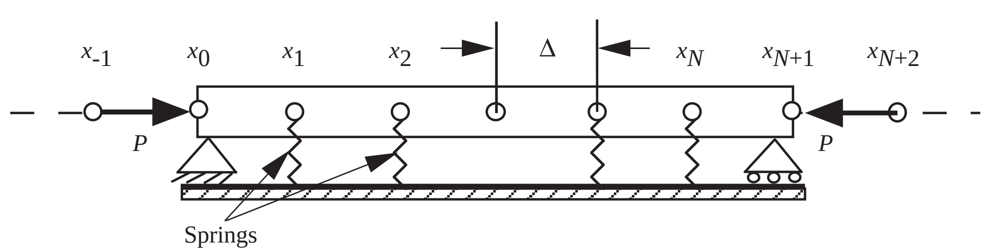


Figure 4. Finite-difference grid with uniform spacing.

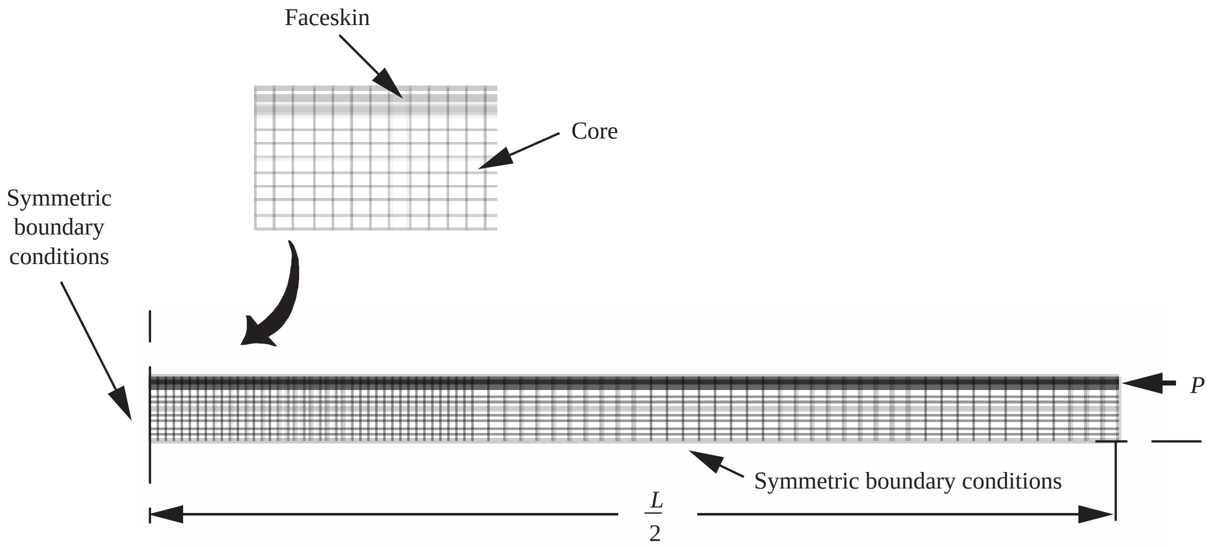


Figure 5. Two-dimensional plane-strain finite-element model.

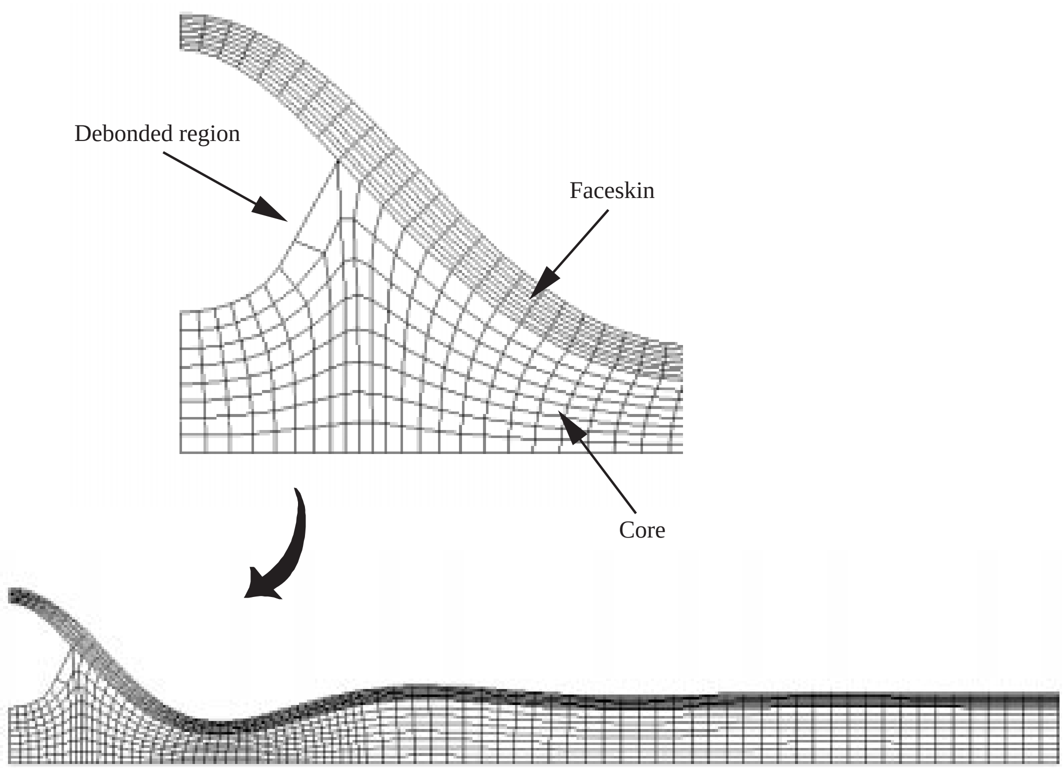


Figure 6. Local buckled region of debonded faceskin of sandwich panel strip.

based on a Mindlin/Reissner shell formulation. Eight layers of 4-noded quadrilateral finite elements are used to model the faceskin and sandwich foam regions. Duplicate nodes for the faceskin and core at their common boundary along the debonded region created the debonded region between the faceskin and core. (See fig. 6.) These duplicate nodes are allowed to translate independently.

A linear stability analysis was performed using NASA Langley Research Center's COmputational MEchanics Testbed (COMET) code (ref. 12) to compute the buckling load of the debonded panel. The linear elastic stability analysis in COMET is formulated using the concept of adjacent equilibrium. Buckling occurs when the membrane strain energy is converted to bending strain energy in the adjacent equilibrium configuration.

The linear elastic stability analysis reduces to an eigenvalue analysis to find the bifurcation from the membrane state to the bending state. The eigenvalue satisfies

$$\mathbf{K}\phi_i + \lambda_i \mathbf{K}_g(\sigma)\phi_i = \mathbf{0} \qquad (i = 1, 2, \ldots) \tag{21}$$

where

$\mathbf{K}$	assembled linear elastic stiffness matrix
$\mathbf{K}_g(\sigma)$	assembled geometric stiffness matrix
ϕ_i	ith eigenvector or mode shape
λ_i	ith eigenvector or buckling load factor

Results and Discussion

Three analytical methods were used to calculate the buckling load of the debonded sandwich panel strip, which consisted of an aluminum faceskin and a foam core. Parametric studies were performed to study how changes in debond length, core moduli, and Poisson's ratios affect the buckling load. Tables 1 and 2 summarize the data used in the analyses for the Rayleigh-Ritz and finite-difference methods and the plane-strain finite-element analyses. The subscripts (i, j = 1, 2) for E_{bij} and E_{fij} are along the x- and z-directions, respectively. In general, the aluminum faceskin and foam core are treated as isotropic materials. However, to simulate the Winkler foundation theory using finite-element analysis, the Poisson's ratios and the foam longitudinal stiffness (x-direction) were given reduced constitutive properties. The Poisson's ratios of the faceskin and core, ν_b and ν_f, were set to 0, and Young's modulus of the sandwich foam, E_{f11}, was set to approximately 0 (10^{-4}) psi. The reduction of constitutive properties allowed evaluation of the effects of the simplifying assumptions of the elastic foundation approach. In addition, the tests evaluated the effects of transverse shear.

Table 1. Geometric and Material Properties

Length of sandwich beam L, in.	30.0
Young's modulus of beam faceskin E_b, psi	10.0×10^6
Elastic foundation modulus K, psi	Variable from 0 to 12500
Thickness of faceskin t_b, in.	0.2
Thickness of core t_f, in.	0.8

Table 2. Constituent Properties of Finite-Element Model

(a) Set 1

Aluminum faceskin (isotropic):	
$E_{b11} = E_{b22} = E_b$, psi	10.0×10^6
Poisson ratio, ν_b	0.30
Shear modulus, G_b	$\frac{E_b}{2(1+\nu_b)}$
Core (isotropic):	
$E_{f11} = E_{f22} = E_f$, psi	Variable from 0 to 10000
Poisson ratio, ν_f	0.30
Shear modulus, G_f	$\frac{E_f}{2(1+\nu_f)}$

(b) Set 2

Aluminum faceskin (reduced):	
Poisson ratio, ν_b	0
Core (reduced):	
E_{f11}, psi	10^{-4}
Poisson ratio, ν_f	0

Effect of Sandwich Foam Stiffness on Buckling Load

Rayleigh-Ritz method. The Rayleigh-Ritz method was used to perform a parametric study of variations in the foundation stiffness of the core. The dimensionless foundation stiffness F varied from 0 (no core stiffness) to a typical foam core stiffness of 30.0 (K = 12 500 psi) for a specific value of the debond ratio μ. The debond length was fixed at 2.0 in., or μ = 0.067 in. The results from this study are presented in terms of the relative buckling load P_{cr}/P_E where P_E denotes the Euler buckling load ($\pi^2 E_b I/L^2$) of the debonded faceskin.

A plot of P_{cr}/P_E versus F in figure 7 shows the convergence of the Rayleigh-Ritz method as the number of shape functions is increased. As expected, the buckling load of the debonded sandwich panel strip increases as the foundation stiffness increases. Twenty-five terms in the Fourier sine series were sufficient to obtain a converged solution.

Finite-difference method. The finite-difference method was also used to predict the buckling load. Figure 8 illustrates the convergence of buckling load as the number of gridpoints is increased when using the finite-difference method. Because the finite-difference method discretely models the elastic foundation as a series of closely spaced springs, a large number of gridpoints are required for a convergent solution. As a result, convergence is slower than the Rayleigh-Ritz method. In addition, the solution procedure of this method was inefficient because the buckling load could not be solved for directly as in the Rayleigh-Ritz method. Instead, the buckling load was determined by using successive iterations of the bisection method until a root of the polynomial equation was found (ref. 14). A change of sign of the determinant of the coefficients in the algebraic equations indicates a root (buckling load) between two successive iterations. Due to the length of the computational time for this analysis, 360 gridpoints were used as the final solution despite the lack of full convergence.

Finite-element method. The 2-D plane-strain finite-element model shown in figure 5 was analyzed using 4-noded quadrilateral elements. The finite-element model consisted of 1280 elements and 1385 nodes for a symmetric model. Duplicate nodes are used to model the debonded region. Two eigenvalue analyses were performed using this model. In the first analysis, the aluminum faceskin and foam core were assumed to be isotropic with the properties shown in table 2. The finite-element results from the isotropic case (at F = 30.0) are about 20 percent higher than those from the Rayleigh-Ritz and finite-difference analyses, as shown in figure 9. This difference increases as the foundation stiffness increases.

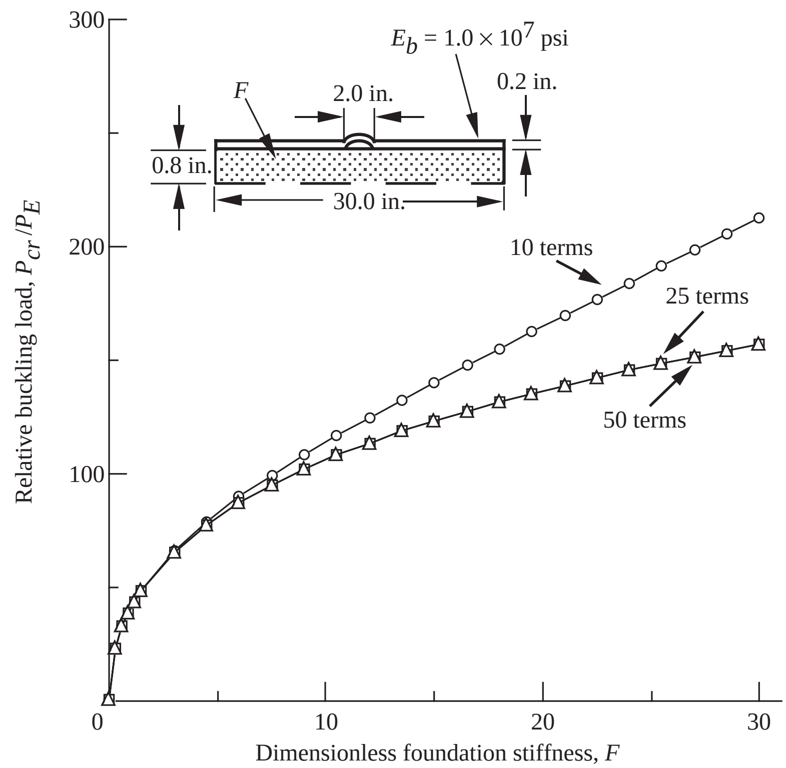


Figure 7. Convergence of relative buckling load using Rayleigh-Ritz method.

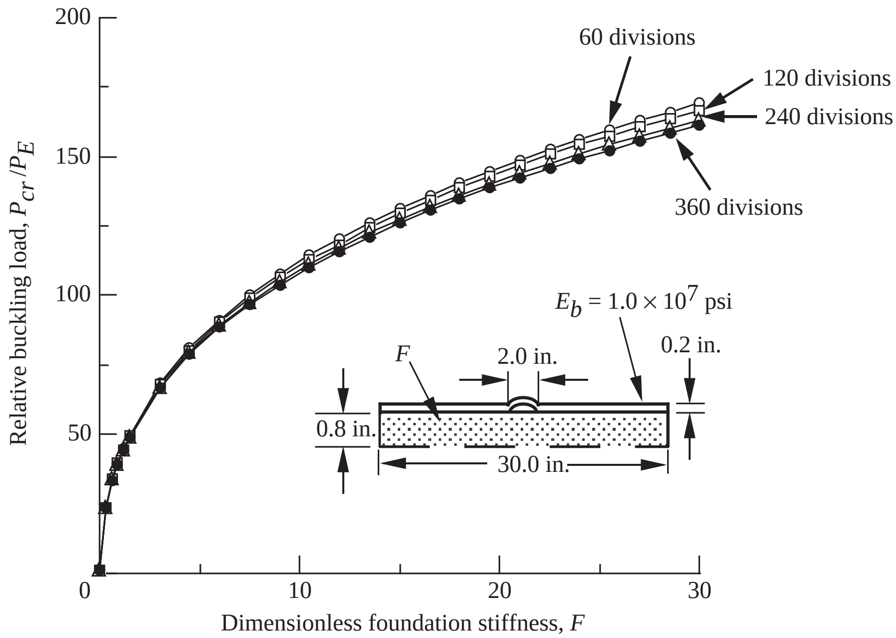


Figure 8. Convergence of relative buckling load using finite-difference method.

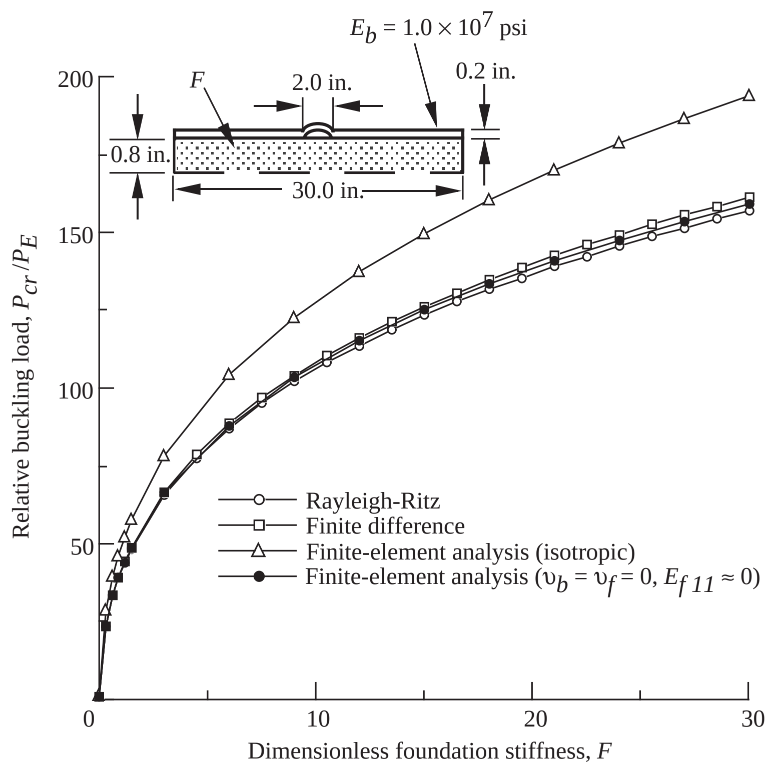


Figure 9. Comparisons of relative buckling load results from different methods for variations in foundation stiffness.

This discrepancy may have been induced by modeling the core material as a Winkler foundation in which the elastic springs represent the core. To verify that the discrepancy is caused by the simplifying assumptions of the elastic foundation approach, a second finite-element analysis was performed with reduced constituent properties. The Poisson's ratios of the aluminum faceskin and foam core, ν_b and ν_f, were set to 0, and Young's modulus of the foam, E_{f11}, was set to approximately 0 (10^{-4}) psi. This reduction of the material properties eliminates the effects of the Poisson's ratios and the longitudinal stiffness of the core. Figure 9 shows a comparison of the second set of finite-element results with the Rayleigh-Ritz and finite-difference methods. Excellent agreement is observed between the finite-element analysis using the reduced material properties and the results from the Rayleigh-Ritz and finite-difference methods.

Effect of Debond Length on Buckling Load

In a second parametric study, the debond ratio μ was varied from 0 (fully bonded) to 1 (fully debonded) for a fixed value of $F = 30.0$ ($K = 12500$ psi). The limiting value of $\mu = 0$ yields the buckling load for a sandwich panel strip with no debonds, and a value of $\mu = 1$ yields the Euler buckling load P_E for a fully debonded faceskin. The results are presented in figure 10 in terms of the normalized buckling load P_{cr}/P_{FULL}, which is the ratio of the buckling load divided by the buckling load of a beam fully supported by an elastic foundation (ref. 10):

$$P_{FULL} = n^2 \frac{\pi^2 E_b I}{L^2} + \frac{kL^2}{n^2 \pi^2} \qquad (n = 1, 2, \ldots) \quad (22)$$

where n is the number of half sine waves required to give the lowest buckling load, depending upon the parameters.

Fifty terms in the Rayleigh-Ritz method and 360 gridpoints in the finite-difference method were used in this study. In the finite-element analysis, two sets of constituent properties were again used. Set 1 used isotropic properties and Set 2 used reduced constituent properties ($\nu_{f12} = \nu_{b12} = 0$, $E_{f11} \approx 0$). The results in figure 10 show that the normalized buckling load decreases as μ increases, indicating that the buckling load for the unsupported skin decreases as the debonded length increases. The results from the isotropic plane-strain finite-element analysis are about 10–20 percent higher (depending on the debond length) than those predicted by the Rayleigh-Ritz and finite-difference methods. This suggests that the simplifying assumptions on Poisson's ratio and the longitudinal stiffness of the core can substantially affect buckling load predictions. However, excellent agreement is observed once these properties are reduced.

Effect of Transverse Shear on Buckling Load

Transverse shear effects were found to be insignificant for the studies performed in this paper. From equation (4.76) in reference (ref. 10), the buckled fully bonded faceskin is calculated to have a wavelength of 2.68 in. for the largest K considered in this study (12500 psi). This corresponds to a l_d/t_b ratio of 13.4, which is estimated to contribute less than 2-percent error for the buckling load of the debonded faceskin using reference (ref. 15). Therefore, transverse shear effects were not considered in the Rayleigh-Ritz and finite-difference analyses.

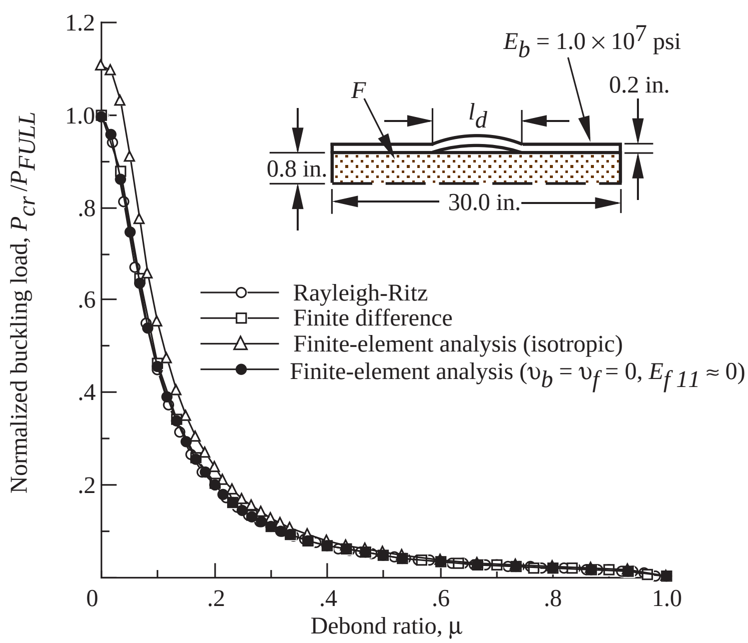


Figure 10. Comparisons of normalized buckling load results from different methods for various debond ratios and $F = 30.0$.

Conclusions

Three numerical methods were used to analyze the buckling of a debonded aluminum faceskin on a sandwich panel with a foam core. The Rayleigh-Ritz and finite-difference methods were used to study a 1-D beam-on-elastic-foundation model that represents a strip of a sandwich panel with a through-the-width debond. The results were compared with the buckling loads calculated from a 2-D finite-element analysis using 4-noded quadrilateral plane-strain elements. Two parametric studies were performed to evaluate the effects of the elastic foundation stiffness and debond length. Results from the three analytical methods were compared and discussed. As expected, the results showed that the buckling load increases with increasing core foundation stiffness and decreases with increasing debond length.

The Rayleigh-Ritz method was shown to be superior to the finite-difference method in obtaining the buckling loads because it requires less computational time and exhibits fast convergence. However, both the Rayleigh-Ritz and finite-difference methods assumed a Winkler core foundation and, thus, ignored the Poisson's ratio and longitudinal stiffness effects of the core material. Results from the Rayleigh-Ritz method and the finite-difference method, which are based on the elastic foundation approach, show lower buckling loads than those predicted by the 2-D plane-strain finite-element analysis, which models the sandwich foam core as an isotropic material. When the core is forced to behave as a Winkler foundation so that the Poisson's ratios of the core are set to 0 and its longitudinal stiffness is reduced to be infinitesimally small, the finite-element analysis agrees with the results from the Rayleigh-Ritz and finite-difference methods. Therefore, to accurately predict the buckling load of a debonded sandwich panel with sandwich foam cores, a finite-element model that includes the Poisson's ratio and longitudinal core stiffness should be used.

NASA Langley Research Center
Hampton, VA 23681-0001
October 16, 1995

References

1. Plantema, Frederik J.: *Sandwich Construction. The Bending and Buckling of Sandwich Beams, Plates, and Shells*. John Wiley & Sons, Inc., 1966.
2. Hwu, Chyanbin; and Hu, Jian S.: Buckling and Postbuckling of Delaminated Composite Sandwich Beams. *AIAA J.*, vol. 30, no. 7, July 1992, pp. 1901–1909.
3. Webster, John D.: *Flaw Criticality of Circular Disbond Defects in Compressive Laminates—1980–81 Interim Report*. NASA CR-164830, June 1981.
4. Chai, Herzl; Babcock, Charles D.; and Knauss, Wolfgang G.: One Dimensional Modelling of Failure in Laminated Plates by Delamination Buckling. *Int. J. Solids & Struct.*, vol. 17, no. 11, 1981, pp. 1069–1083.
5. Chai, Herzl; and Babcock, Charles D.: Two-Dimensional Modelling of Compressive Failure in Delaminated Laminates. *J. Compos. Mater.*, vol. 19, Jan. 1985, pp. 67–98.
6. Somers, M.; Weller, T.; and Abramovich, H.: Influence of Predetermined Delaminations on Buckling and Postbuckling Behavior of Composite Sandwich Beams. *Compos. Struct.*, vol. 17, no. 4, 1991, pp. 295–329.
7. Vizzini, A. J.; and Lagace, P. A.: The Buckling of a Delaminated Sublaminate on an Elastic Foundation. *J. Compos. Mater.*, vol. 21, Dec. 1987, pp. 1106–1117.
8. Kim, W. C.; and Dharan, C. K. H.: Facesheet Debonding Criteria for Composite Sandwich Panels Under In-Plane Compression. *Eng. Fract. Mech.*, vol. 42, no. 4, July 1992, pp. 643–652.
9. Brush, Don O.; and Almroth, Bo O.: *Buckling of Bars, Plates, and Shells*. McGraw-Hill, Inc., 1975.
10. Chen, Wai-Fah; and Atsuta, Toshio: *Theory of Beam Columns.* Volume 1—In-Plane Behavior and Design. McGraw-Hill, Inc., 1976.
11. Langhaar, Henry L.: *Energy Methods in Applied Mechanics.* John Wiley & Sons, Inc., 1962.
12. Stewart, Caroline B.: *The Computational Structural Mechanics Testbed User's Manual*. NASA TM-100644, 1989.
13. Park, K. C.; and Stanley, G. M.: A Curved C^0 Shell Element Based on Assumed Natural-Coordinate Strains. *J. Appl. Mech.*, vol. 53, no. 2, June 1986, pp. 278–290.
14. Barnard, David T.; and Skillicorn, David B.: *Effective Fortran 77 for Engineers & Scientists*. William C. Brown Publ., 1988.
15. Timoshenko, S.: *Theory of Elastic Stability.* McGraw-Hill, Inc., 1936.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
December 1995

3. REPORT TYPE AND DATES COVERED
Technical Memorandum

4. TITLE AND SUBTITLE
Buckling Analysis of Debonded Sandwich Panel Under Compression

5. FUNDING NUMBERS
WU 510-02-12-01

6. AUTHOR(S)
David W. Sleight and John T. Wang

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Langley Research Center
Hampton, VA 23681-0001

8. PERFORMING ORGANIZATION REPORT NUMBER
L-17476

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA TM-4701

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified–Unlimited
Subject Category 01
Availability: NASA CASI (301) 621-0390

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
A sandwich panel with initial through-the-width debonds is analyzed to study the buckling of its faceskin when subject to an in-plane compressive load. The debonded faceskin is modeled as a beam on a Winkler elastic foundation in which the springs of the elastic foundation represent the sandwich foam. The Rayleigh-Ritz and finite-difference methods are used to predict the critical buckling load for various debond lengths and stiffnesses of the sandwich foam. The accuracy of the methods is assessed with a plane-strain finite-element analysis. Results indicate that the elastic foundation approach underpredicts buckling loads for sandwich panels with isotropic foam cores.

14. SUBJECT TERMS
Sandwich structures; Prediction analysis techniques; Buckling; Delaminating; Elastic properties

15. NUMBER OF PAGES
11

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT
Unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT
Unclassified

20. LIMITATION OF ABSTRACT

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102