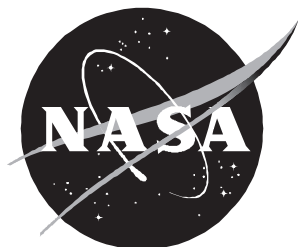# Parameterizations of Pion Energy Spectrum in Nucleon-Nucleon Collisions

Francis A. Cucinotta and John W. Wilson
Langley Research Center, Hampton, Virginia

John W. Norbury
University of Wisconsin, Milwaukee, Wisconsin

The NASA STI Program Office . . . in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:
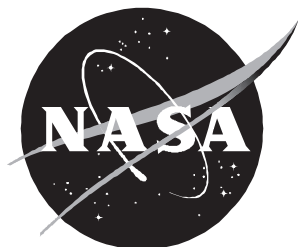
- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counter-part or peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.

- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.

- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that help round out the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at *http://www.sti.nasa.gov*

- Email your question via the Internet to help@sti.nasa.gov

- Fax your question to the NASA Access Help Desk at (301) 621-0134

- Phone the NASA Access Help Desk at (301) 621-0390

- Write to:
  NASA Access Help Desk
  NASA Center for AeroSpace Information
  7121 Standard Drive
  Hanover, MD 21076-1320

Parameterizations of Pion Energy Spectrum in Nucleon-Nucleon Collisions

Francis A. Cucinotta and John W. Wilson
Langley Research Center, Hampton, Virginia

John W. Norbury
University of Wisconsin, Milwaukee, Wisconsin

Abstract

The effects of pion (π) production are expected to play an important role in radiation exposures in the upper atmosphere or on the Martian surface. Nuclear databases for describing pion production are developed for radiation transport codes to support these studies. We analyze the secondary energy spectrum of pions produced in nucleon-nucleon (NN) collisions in the relativistic one-pion exchange model. Parametric formulas of the isospin cross sections for one-pion production channels are discussed and are used to renormalize the model spectrum. Energy spectra for the deuteron related channels (NN $\rightarrow d\pi$) are also described.

1. Introduction

A convenient representation of the differential cross section in energy of particles created in the interactions of space and atmospheric radiation with materials is required for radiation transport computer codes to be efficient for engineering design and risk assessment (refs. 1 through 3). In high-energy reactions, an abundance of mesons is produced through the nuclear force. High-energy reactions increase in relative importance in the upper atmosphere due to the Earth's magnetic field, reducing lower energy ion components which have insufficient energy to overcome the threshold for meson production. The one-pion production channels dominate the meson production for energies up to about 1.5 GeV/amu and represent an important contribution at higher energies. The energy region below 1.5 GeV/amu extends above the galactic cosmic ray (GCR) peak at 0.2 to 0.6 GeV/amu. Also, most of the energy range of particles seen in solar particle events (SPE) are dominated by the one-pion exchange interactions. The scattering of high-energy protons or neutrons on hydrogen is important because protons and neutrons represent over 90 percent of the particle flux in materials and also because hydrogenous materials in tissue and material structures are important. The description of the one-pion production mechanism in nucleon-nucleon (NN) collisions is also needed for modeling the pion production mechanism in inclusive proton and neutron collisions on target atoms in applying reaction theory or in Monte Carlo simulations (refs. 4 and 5).

In this report, we describe the calculation of the secondary energy spectrum in proton-proton (pp) and neutron-proton (np) reactions. The one-pion production channels are modeled by using the relativistic one-pion exchange (OPE) model. Parametric models for the isospin components in NN reactions and πN reactions are discussed. The one-pion channels discussed here can be appended with parameterizations of high-energy models (refs. 6 through 8) to provide the energy spectrum for inclusive pion production in NN collisions at overall energies of interest for space radiation studies.

2. One-Pion Production Cross Sections

The cross sections for production of a single pion in nucleon-nucleon (NN) collisions may be written in terms of four independent cross sections by applying isospin conservation. For a transition from an initial isospin state I_i to a final isospin state I_f the cross section is denoted $\sigma_{I_i I_f}$. For formation of a deuteron d in the final state, a superscript d is used. In table 1 we list the isospin and masses of the particles we consider. In table 2 we list the relationship between the four independent cross sections and the various reaction channels in NN collisions. Various authors have considered parameterizations of these cross sections: σ_{10}, σ_{01}, σ_{11}, and σ_{10}^d. The fits of Wilson and Chun (1988) are useful because they extend over all energies. The work of VerWest and Arndt (ref. 9) is significant because they have

reconsidered discrepancies in older data sets, resulting in a large reduction in the isospin zero cross section σ_{01} over previous estimates and are in good agreement with the more recent measurements (ref. 9). The formula of VerWest and Arndt (ref. 9) for σ_{10}^d is (in units where $\hbar = c = 1$)

$$\sigma_{10}^d(s) = \frac{\pi}{2p^2}\alpha\left(\frac{p_r}{p_o}\right)^\beta \frac{m_o^2\Gamma^2}{\left(s_{\pi N} - m_o^2\right)^2 + m_o^2\Gamma^2} \tag{2.1}$$

where

$$p^2 = s/4 - m_N^2$$

$$s = 4m_N^2 + 2m_N T_L$$

$$s_{\pi N} = (\sqrt{s} - m_N)^2$$

$$p_r^2(s) = \frac{[s - (m_d - m_\pi)^2][s - (m_d + m_\pi)^2]}{4s}$$

$$p_o^2 = s_o/4 - m_N^2$$

$$s_o = (m_N + m_o)^2$$

with m_N the nucleon mass, $m_N = 0.939$ GeV, m_π the pion mass, and $m_\pi = 0.138$ GeV. The formula used in reference 9 for $\sigma_{I_i I_f}$ is

$$\sigma_\pi(s) = \frac{\pi}{2p^2}\alpha\left(\frac{p_r}{p_o}\right)^\beta \frac{m_o^2\Gamma^2\left(q/q_o^3\right)}{\left(s^* - m_o^2\right)^2 + m_o^2\Gamma^2} \tag{2.2}$$

where $s^* = \langle M \rangle^2$,

$$p_r^2(s) = \frac{[s - (m_N - \langle M \rangle)^2][s - (m_N + \langle M \rangle)^2]}{4s}$$

$$q_r^2(s^*) = \frac{[s^* - (m_N - m_\pi)^2][s^* - (m_N + m_\pi)^2]}{4s}$$

$$q_o = q\left(m_o^2\right)$$

$$\langle M(s) \rangle = M_o + (\arctan Z_+ - \arctan Z_-)^{-1}\left(\frac{1 + Z_+^2}{1 + Z_-^2}\right)$$

2

where

$$Z_+ = (\sqrt{s} - m_N - M_o)(2/\Gamma_o)$$
$$Z_- = (m_N + m_\pi - M_o)(2/\Gamma_o)$$

with $M_o = 1.22$ GeV and $\Gamma_o = 0.12$ GeV. The parameters of the model are listed in table 3. The formulas of equations (2.1) and (2.2) are valid only for $T_L < 1.5$ GeV.

Wilson and Chun have considered the following parameterizations of the isospin cross sections for all energies:

$$\sigma_{10}(d) = \begin{cases} 2.563\ e^{0.435T_L} - 17.47\ e^{-6.044T_L} & (T_L \leq 0.6\text{ GeV}) \\ 7.531\ e^{2.82T_L} + 44.8\ e^{-5.69T_L} & (0.6 < T_L \leq 1.3\text{ GeV}) \\ 0.22\ e^{0.0885T_L} - 1.96\ e^{-1.754T_L} & (1.3 \leq T_L) \end{cases} \qquad (2.3)$$

$$\sigma_{10}(np) = \frac{36}{T_L^{1.2}}\left[1 - e^{-0.7(T_L - T_{th})^{1.6}}\right] \qquad (2.4)$$

$$\sigma_{01} = \frac{7.2}{T_L^{1.1}}\left[1 - e^{-1.1(T_L - T_{th})^{1.4}}\right]\left[1 - e^{-2(T_L - T_{th})^2}\right] \qquad (2.5)$$

$$\sigma_{11} = \frac{5}{T_L^{0.522}}\left[1 - e^{-3.75(T_L - T_{th})^2}\right] \qquad (2.6)$$

where T_{th} is the single-pion production threshold and T_L is the laboratory nucleon energy.

Comparisons of the fits described above are shown in figure 1. Large discrepancies exist, especially for σ_{01} and σ_{10}. Because the model of VerWest and Arndt (ref. 9) is more accurate at lower energies, we will use this model below 1.3 GeV. The resulting fit is shown in figure 2, and comparisons to data (ref. 10) for π^+ and π^0 production in proton-proton (pp) collisions are shown in figure 3. The result of joining equations (2.1) and (2.2) with equations (2.3) to (2.6) will represent the one-pion production data quite accurately over the energy range of interest for radiation transport codes and will be used to renormalize the model spectra described in the next section.

3. Pion Energy Spectrum

The deuteron production channels are two-body final states and are therefore much easier to parameterize than the other pion production channels. By isospin conservation (neglecting Coulomb effects), we have the relationship

$$\sigma(pn \to \pi^0 d) = \frac{1}{2}\sigma(pp \to \pi^+ d) = \frac{1}{2}\sigma_{10}^d \qquad (3.1)$$

3

The angular distribution is parameterized as

$$\frac{d\sigma_{10}^d}{d\Omega} = \sigma_{10}^d N_{10}^d \left(A_{10}^d + \cos^2\theta - B_{10}^d \cos^4\theta \right) \tag{3.2}$$

where N_{10}^d is a normalization constant and θ is the cm scattering angle. The energy dependent parameters A_{10}^d and B_{10}^d are

$$A_{10}^d = 0.27 \left[1 + 0.13 \cos\left(\frac{T_L}{0.182} \right) \right] \tag{3.3}$$

and

$$B_{10}^d = \begin{cases} 0 & (T_L < 0.4 \text{ GeV}) \\ 0.6 \left\{ 1 - \exp\left[\frac{-(T_L + 0.4)}{0.3} \right] \right\} & (T_L \geq 0.4 \text{ GeV}) \end{cases} \tag{3.4}$$

Comparisons of equations (3.1) to (3.4) to experimental data (refs. 11 and 12) are shown in figure 4(a) for the $\pi^+ d \to pp$ reaction and in figure 4(b) for the $pp \to \pi^+ d$ reaction.

The energy distribution in the laboratory frame of the final deuteron is related to the center of mass (c.m.) angular distribution by

$$\frac{d\sigma}{dT_d} = \frac{2\pi s}{m_N \sqrt{\lambda\left(s, m_N^2, m_N^2\right) \lambda\left(s, m_d^2, m_\pi^2\right)}} \frac{d\sigma_{10}^d}{d\Omega} \tag{3.5}$$

where the function λ is defined

$$\lambda(s, A, B) = (s - A - B)^2 - 4AB \tag{3.6}$$

and s is the Mandelstam variable given by $s = \left(p_{N_1} + p_{N_2} \right)^2$.

In applying equation (3.5), we use

$$\cos\theta = \frac{m_d^2 - m_\pi^2 - 2m_N m_d + 2\bar{E}_\pi \bar{E}_p - 2m_N T_d}{2\bar{p}_p \bar{p}_{\bar{A}}} \tag{3.7}$$

where barred quantities are the c.m. values given by

$$\bar{E}_\pi = \frac{s + m_d^2 - m_\pi^2}{2\sqrt{s}} \tag{3.8}$$

$$\bar{E}_d = \frac{s + m_\pi^2 - m_d^2}{2\sqrt{s}} \tag{3.9}$$

$$\bar{E}_p = \frac{\sqrt{s}}{2} \tag{3.10}$$

The kinematical limits on the kinetic energy of the deuteron T_d are found from equation (3.7) by observing that $|\cos\theta| \leq 1$.

The energy distribution in the laboratory reference frame for the pion is

$$\frac{d\sigma}{dT_\pi} = \frac{2\pi s}{m_N \sqrt{\lambda\left(s, m_N^2, m_N^2\right)\lambda\left(s, m_d^2, m_\pi^2\right)}} \frac{d\sigma_{10}^d}{d\Omega} \tag{3.11}$$

with

$$\cos\theta = \frac{m_\pi^2 - m_d^2 - 2m_N m_\pi + 2\bar{E}_d\bar{E}_\pi - 2m_N T_\pi}{2\bar{P}_p\bar{P}_d} \tag{3.12}$$

and a similar expression is found for the deuteron spectrum. Calculations of energy spectra of secondary pions and deuterons for several beam energies are shown in figures 5 and 6, respectively.

The NN $\rightarrow$ NNπ reactions are assumed to proceed through the formation and decay of the Δ resonance. The mechanism is described by figure 7(a). The Δ forms an isospin quartet ($I = 3/2$), and it is useful to consider the coupling of the πN system wave functions in isospin space to understand the Δ decay properties. Denoting the nucleon isospin by I_N and the pions by I_π, we have

$$|N\pi\rangle = \sum_{I, I_Z} |II_Z\rangle\langle II_Z | I_N I_{N_Z} I_\pi I_{\pi_Z}\rangle \tag{3.13}$$

Equation (3.13) is used to obtain the components of the Δ wave function with the result

$$|\Delta^{++}\rangle = |\pi^+ p\rangle \tag{3.14a}$$

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}}[|\pi^+ n\rangle + \sqrt{2}\,|\pi^o p\rangle] \tag{3.14b}$$

5

$$|\Delta^o\rangle = \frac{1}{\sqrt{3}}[\sqrt{2}\,|\pi^o n\rangle + |\pi^- p\rangle] \tag{3.14c}$$

$$|\Delta^-\rangle = |\pi^- p\rangle \tag{3.14d}$$

Branching ratios for the formation of various pion species in NN collisions are obtained by using equations (3.14).

The invariant differential cross section for $NN \to N\Delta$ has been evaluated in the relativistic one-pion exchange (OPE) model. We use this model and the assumption of isotropic decay of the Δ in its rest frame to obtain the momentum distribution and energy spectrum of pions and nucleons in the $NN \to NN\pi$ channels. The Mandelstam variables for the reaction are defined in terms of the four momentum vectors of the various particles defined in figure 7 and given by

$$s = (K_1 + K_2)^2 \tag{3.15}$$

$$t = (K_1 - K_3)^2 \tag{3.16}$$

$$u = (K_2 - K_3)^2 \tag{3.17}$$

The cross-section distribution in t is written in terms of the matrix element M as

$$\frac{d\sigma}{dt} = \frac{1}{64\pi}|M|^2 \frac{1}{4I_F^2} \tag{3.18}$$

where

$$I_F = \sqrt{(K_1 K_2)^2 - m_N^2} \tag{3.19}$$

The matrix element M is decomposed as

$$|M|^2 = |M_{\mathrm{dir}}|^2 + |M_{\mathrm{ex}}|^2 + |M_{\mathrm{INT}}|^2 \tag{3.20}$$

The direct amplitude is given by (ref. 13)

$$|M_{\mathrm{dir}}|^2 = \left(\frac{f_\pi f_\pi^*}{m_\pi}\right)^2 \frac{F^4(t)}{\left(t - m_\pi^2\right)^2} \frac{t[t - (m_\Delta - m_N)^2]\,[(m_\Delta + m_N)^2 - t]^2}{3m_\Delta^2} \tag{3.21}$$

where f_π and f_π^* are the coupling constants with values 1.008 and 2.202, respectively, and m_π and m_Δ are the pion-fixed Δ mass with values 0.139 GeV and 1.232 GeV, respectively. In equation (3.21), $F(t)$ is the form factor for the off-shell meson which is parameterized as

6

$$F(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t} \tag{3.22}$$

where the value of the parameter Λ will be discussed below. The exchange term in equation (3.20) is equivalent in form to equation (3.21) with the replacement $(u \leftrightarrow t)$.

The interference term is given by

$$\frac{1}{4}|M_{\text{INT}}|^2 = \left(\frac{f_\pi \, f_\pi^*}{m_\pi}\right) \frac{F^2(t)F^2(u)}{\left(t - m_\pi^2\right)\left(u - m_\pi^2\right)} \frac{1}{6m_\Delta^2}$$

$$\times \left\{ \left[tu + \left(m_\Delta^2 - m_N^2\right)(t + u) - \left(m_\Delta^4 + m_N^4\right) \right] \left[tu + m_N(m_N + m_\Delta)\left(m_\Delta^2 - m_N^2\right) \right] \right.$$

$$\left. + \left[tu - \left(m_\Delta^2 - m_N^2\right)(t + u) + (m_N + m_\Delta)^4 \right] \left[tu - m_N(m_\Delta - m_N)\left(m_\Delta^2 - m_N^2\right) \right] \right\} \tag{3.23}$$

In order to account for the finite width of the Δ resonance, a mass distribution $\rho(\mu^2)$ is introduced through

$$\frac{d\sigma}{dt \, d\mu^2} = A(s, t, u)\rho(\mu^2) \tag{3.24}$$

where $\mu^2 = (K_4 + K_\pi)^2$ and $A(s, t, u)$ is the invariant cross section of equation (3.18) with the fixed Δ mass m_Δ replaced by μ. The mass distribution is parameterized in terms of the elastic pion-nucleon cross section and the Δ width as (ref. 13)

$$\rho(\mu^2) = \frac{K_\Delta^2 \sigma_{\pi N}}{8\pi^2 m_\Delta \Gamma} \tag{3.25}$$

where K_Δ is given by

$$K_\Delta(\mu^2, m_\pi^2) = \sqrt{\frac{\mu^2 + m_N^2 - m_\pi^2}{4\mu^2} - m_N^2} \tag{3.26}$$

The width is parameterized as

$$\Gamma = \Gamma_o \left[\frac{K_\Delta\left(\mu^2, m_\pi^2\right)}{K_\Delta\left(m_\Delta^2, m_\pi^2\right)}\right]^3 Z\left(\mu^2, m_\pi^2\right) \tag{3.27}$$

with the form factor Z given by

$$Z(\mu^2, m_\pi^2) = \frac{K_\Delta^2\left(m_\Delta^2, m_\pi^2\right) + \kappa^2}{K_\Delta^2\left(\mu^2, m_\pi^2\right) + \kappa^2} \tag{3.28}$$

and parameter values $\Gamma_o = 0.12$ GeV, and $\kappa = 0.2$ GeV. By neglecting Coulomb effects, the shape of the secondary energy spectrum for the components of the isospin triplet of pions is largely determined by the πN cross section in equation (3.25), with the remaining kinematic factors in equations (3.18) to (3.28) being identical for each pion species. The description of the πN cross sections is given in the appendix.

The decay of the Δ ($\Delta \to N\pi$) is assumed to be isotropic in the Δ rest frame such that the pion distribution from the decay is

$$\frac{d\sigma}{dt\, d\mu^2 d\Omega_\pi^*} = \frac{1}{4\pi} A(s, t, u)\rho(\mu^2) \tag{3.29}$$

and the nucleon distribution from the decay is

$$\frac{d\sigma}{dt\, d\mu^2 d\Omega_N^*} = \frac{1}{4\pi} A(s, t, u)\rho(\mu^2) \tag{3.30}$$

where starred quantities are in the Δ rest frame. A second contribution to the nucleon spectrum comes from the nondecaying nucleon line in figure 7, as discussed below. The Δ mass is found as

$$\mu^2 = \left(E_4^* + E_\pi^*\right)^2 \tag{3.31}$$

and using

$$d\mu^2 = \frac{\partial\mu^2}{\partial E_\pi^*}\, dE_\pi^* \tag{3.32}$$

allows the invariant momentum distribution of the pion to be written

$$E_\pi \frac{d\sigma}{d\bar{p}_\pi} = \frac{1}{4\pi^2} \int d\bar{\Omega}_3 \left(\frac{\bar{p}_1\bar{p}_3}{p_\pi^*}\right)\frac{\partial\mu^2}{\partial E_\pi^*} A(s, t, u)\rho(\mu^2) \tag{3.33}$$

8

with a similar expression for the decay nucleon spectrum. The energy spectrum of the pion in the laboratory system is then

$$\frac{d\sigma}{dE_\pi} = \frac{1}{4\pi^2} \int d\bar{\Omega}_3 \, d\Omega_\pi p_\pi \left(\frac{\bar{p}_1 \bar{p}_3}{p_\pi^*}\right) \frac{\partial \mu^2}{\partial E_\pi^*} A(s, t, u) \rho(\mu^2) \tag{3.34}$$

The laboratory energy spectrum of the decay nucleon is

$$\frac{d\sigma^D}{dE_N} = \frac{1}{4\pi^2} \int d\bar{\Omega}_3 \, d\Omega_N p_N \left(\frac{\bar{p}_1 \bar{p}_3}{p_N^*}\right) \frac{\partial \mu^2}{\partial E_N^*} A(s, t, u) \rho(\mu^2) \tag{3.35}$$

The energy spectrum of the recoil nucleon is given by (ref. 14)

$$\frac{d\sigma^R}{dE_N} = \int_o^{\theta_N^{max}} d(\cos \theta_N) \frac{m_N^2 \mu^2}{\pi m_\Delta^2} \frac{p_N}{p_o} A(s, t, u) \rho(\mu^2) \tag{3.36}$$

where p_o is the laboratory momentum of the incident nucleon.

A comparison of equation (3.24) to experimental data for the $pp \rightarrow n\Delta^{++}$ reaction (refs. 10 and 15) is shown in figure 8, with good agreement found. The parameter Λ in the form factor of equation (3.22) has a strong effect on the calculations. As noted by Jain and Santra (ref. 14), the inclusion of distorted waves effects, the value of Λ chosen, and a value of ~1 GeV provides the best fit to data. Calculations of energy spectrum for π^+ and π^o production in pp collisions and π^+, π^o, π^- production in neutron-proton (np) collisions are shown in figures 9 and 10, respectively. The variations in the $\Delta \rightarrow \pi N$ decay vertices due to the isospin dependence provide only a modest change in shape between the various production channels. Coulomb effects which are not treated herein will provide further dependence on the pion charge.

4. Inclusive Pion Production Spectrum

As the kinetic energy of nucleons increases, the threshold for 2, 3 . . . pion production is reached and so is production of heavier mesons such as the kaon. The threshold energies for several production processes are listed in table 4. The production threshold is also dependent on charge conservation when individual species of mesons in the reaction are produced. Inclusive meson production data have been parameterized in convenient forms by several authors (refs. 6 through 8). The spectrum in one-pion, two-pion, and other channels will be somewhat distinct due to the mechanisms involved. The two-pion production channels have been considered by Sternheimer and Lihdenbaum (ref. 16) using a purely kinematic form of the isobar model. There are two distinct mechanisms for two-pion production which are through the excitation of two Δ's:

$$N + N \rightarrow \Delta_1 + \Delta_2 \rightarrow N + N + \pi + \pi \tag{4.1}$$

or through the excitation of higher mass nucleon resonances:

$$N + N \rightarrow N + N^* \rightarrow N + N + \pi + \pi \tag{4.2}$$

9

where N^* is a nucleon resonance of higher mass than the Δ, which is assumed to decay through the emission of two pions. The mechanisms of equations (4.1) and (4.2) are illustrated in figures 7(b) and 7(c), respectively. The mechanism of equation (4.2) will have a cross-section structure similar to the one-pion model, $NN \rightarrow NN\pi$, described above. However, with $\Delta \rightarrow N\pi$, the vertex is replaced by the $N^* \rightarrow N\pi\pi$ vertex. The mechanism of equation (4.3) will contain the mass distribution of two Δ's with a structure such as

$$\frac{d\sigma}{dt\, d\mu_1^2\, d\mu_2^2} = A(s, t, u)\rho_1\left(\mu_1^2\right)\rho_2\left(\mu_2^2\right) \tag{4.3}$$

thus requiring one additional numerical integral to obtain the pion energy spectrum.

The inclusive pion spectrum can be represented as a sum over the spectrum for each multiplicity at pion as

$$\frac{d\sigma}{dE_\pi} = \frac{d\sigma}{dE_{\pi_1}} + \frac{d\sigma}{dE_{\pi_2}} + \dots \tag{4.4}$$

where $\dfrac{d\sigma}{dE_{\pi_1}}$ represents the spectrum described by equation (3.34). The inclusive spectrum can be repre-

sented by the one-pion channel plus the high-energy model of Schneider, Norbury, and Cucinotta (ref. 7) renormalized to exclude the one-pion contributions, or alternatively, by including the two-pion production channels separately, as described previously.

5. Concluding Remarks

One-pion production channels will dominate pion production for a significant fraction of galactic cosmic ray (GCR) and solar particle event (SPE) exposures in free space, in the upper atmosphere, or on the Martian surface. In this report, the one-pion production cross sections were discussed, and a convenient formula for their numerical representation was found. These models will be appended with high-energy models to span all energies of importance in GCR studies. The formula described here also can be used to model pion and nucleon production spectra in nucleon-nucleus and nucleus-nucleus reactions.

NASA Langley Research Center
Hampton, VA 23681-2199
September 1, 1998

Appendix

Pion-Nucleon Scattering

The πN scattering amplitude is written to represent the pion isospin $\bar{I}_\pi$ and nucleon isospin $\frac{1}{2}\bar{\tau}_N$ as

$$f_{\pi N} = \frac{1}{3}(f_{1/2} + 2f_{3/2}) + \frac{1}{3}(f_{3/2} - f_{1/2})\bar{I}_\pi \cdot \bar{\tau}_N \tag{A1}$$

where $f_{1/2}$ and $f_{3/2}$ are amplitudes for total isospin 1/2 or 3/2, respectively. Introducing the total isospin of the πN system,

$$\bar{T} = \bar{I}_\pi + \frac{1}{2}\bar{\tau}_N \tag{A2}$$

and πN wave functions

$$|TT_Z\rangle = \sum_{I_Z}\langle I_\pi I_Z \tau_N \tau_Z \,|\, TT_Z\rangle \,|\, I_\pi I_Z \tau_N \tau_Z\rangle \tag{A3}$$

and noting

$$\bar{I}_\pi \cdot \bar{\tau}_N = \left(T^2 - \frac{11}{4}\right) \tag{A4}$$

leads to the following relations for elastic scattering:

$$\langle \pi^+ p \,|\, f_{\pi N} \,|\, \pi^+ p\rangle = f_{3/2} \tag{A5}$$

$$\langle \pi^o p \,|\, f_{\pi N} \,|\, \pi^o p\rangle = \frac{1}{3}(f_{1/2} + 2f_{3/2}) \tag{A6}$$

$$\langle \pi^- p \,|\, f_{\pi N} \,|\, \pi^- p\rangle = \frac{1}{3}(f_{3/2} + 2f_{1/2}) \tag{A7}$$

$$\langle \pi^+ n \,|\, f_{\pi N} \,|\, \pi^+ n\rangle = \frac{1}{3}(f_{3/2} + 2f_{1/2}) \tag{A8}$$

$$\langle \pi^o n \,|\, f_{\pi N} \,|\, \pi^o n\rangle = \frac{1}{3}(f_{1/2} + 2f_{3/2}) \tag{A9}$$

$$\langle \pi^- n \,|\, f_{\pi N} \,|\, \pi^- n\rangle = f_{3/2} \tag{A10}$$

with similar relations found for charge-exchange matrix elements.

The total cross sections can be found from the optical theorem. Numerous measurements exist for the total $\pi^+ p$ and $\pi^- p$ reactions. From equations (A5) to (A10), we can find solutions for the other πN collision pairs (neglecting Coulomb effects) by using equations (A11) through (A14).

$$\sigma^T_{\pi^o p} = \frac{1}{2}\left[\sigma^T_{\pi^+ p} + \sigma^T_{\pi^- p} - \sigma_{EX}(\pi^- p)\right] \tag{A11}$$

$$\sigma^T_{\pi^+ n} = \sigma^T_{\pi^- p} \tag{A12}$$

$$\sigma^T_{\pi^o n} = \sigma^T_{\pi^o p} \tag{A13}$$

$$\sigma^T_{\pi^- n} = \sigma^T_{\pi^+ p} \tag{A14}$$

References

1. Wilson, John W.; Townsend, Lawrence W.; Schimmerling, Walter; Khandelwal, Govind S.; Khan, Ferdous; Nealy, John E.; Cucinotta, Francis A.; Simonsen, Lisa C.; Shinn, Judy L.; and Norbury, John W.: *Transport Methods and Interactions for Space Radiations*. NASA RP-1257, 1991.

2. Cucinotta, Francis A.: *Calculations of Cosmic-Ray Helium Transport in Shielding Materials*. NASA TP-3354, 1993.

3. Cucinotta, F. A.; Townsend, L. W.; Wilson, J. W.; Shinn, J. L.; Badhwar, G. D.; and Dubey, R. R.: Light Ion Components of the Galactic Cosmic Rays: Nuclear Interactions and Transport Theory. *Adv. Space Res.*, vol. 17, no. 2, 1996, pp. 77–86.

4. Bertini, H. W.: Nonelastic Interactions of Nucleons and Pi Mesons With Complex Nuclei at Energies Below 3 GeV. *Phys. Rev. C*, third ser., vol. 6, Aug. 1972, pp. 631–659.

5. Lemaire, M. C.: Medium Effects in Pion Production. *Nucl. Phys. A,* vol. 489, no. 4, 1988, pp. 781–802.

6. Badhwar, G. D.; Golden, R. L.; and Stephens, S. A.: Analytic Representation of the Proton-Proton and Proton-Nucleus Cross-Sections and Its Application to the Sea-Level Spectrum and Charge Ratio of Muons. *Phys. Rev. D*, vol. 15, Feb. 1977, pp. 820–831.

7. Schneider, John P.; Norbury, John W.; and Cucinotta, Frank A.: Parameterization of Spectral Distributions for Pion and Kaon Production in Proton-Proton Collisions. *Astrophys. J. Suppl. Ser.*, vol. 97, no. 2, Apr. 1995, pp. 571–574.

8. Choudhur, R. K.; Bhattach, S.; and Bhattach, D. P.: Lepton-Hadron Relation and the Inclusive Low-PT Charged Secondaries at ISR Energies in *pp* Scattering. *Nuov. Cim. A*, vol. 50, no. 1, 1979, pp. 133–154.

9. VerWest, B. J.; and Arndt, R. A.: *NN* Single Pion Production Cross Sections Below 1500 MeV. *Phys. Rev. C,* vol. 25, no. 4, 1982, pp. 1979–1985.

10. Kubota, Y.; Koiso, H.; Sai, F.; Sakamoto, S.; and Yamamoto, S. S.: Measurement of the *pp* Cross Sections in the Momentum Range 0.9–2.0 GeV/c. *Nucl. Phys. A*, vol. 386, no. 2, 1982, pp. 571–588.

11. Richard-Serre, C.: *Study of the Reaction Positive Pion Plus Deuteron Yields Proton Plus Proton for Pion Energy Range Between 143 and 264 MeV*. CERN-68-40, European Org. Nucl. Res., Dec. 1968.

12. Heinz, R. M.; Overseth, O. E.; Pellett, D. E.; and Perl, M. L.: Differential Cross Sections for $p + p \to d + \pi^+$ From 1 to 3 BeV. *Phys. Rev.*, vol. 167, no. 5, 1968, pp. 1232–1239.

13. Dmitriev, V.; Sushkov, O.; and Gaarde, C.: Δ-Formation in the ^{1}H (^{3}He, ^{3}H) Δ^{++} Reaction at Intermediate Energies. *Nucl. Phys. A*, vol. 459, nos. 3–4, 1986, pp. 503–524.

14. Jain, B. K.; and Santra, A. B.: Study of the p(p, n) Δ^{++} Reaction in Nuclei. *Nucl. Phys.*, vol. A519, 1990, pp. 697–720.

15. Wicklund, A. B.; Arenton, M. W.; Ayres, D. S.; Diebold, R.; Kramer, S. L.; May, E. N.; Modulman, L. J.; and Sauer, J. R.: Study of the Reaction $pp \to p\pi^+ n$ With Polarized Beams From 3 to 12 GeV/*c*. *Phys. Rev. D*, vol. 34, no. 1, 1986, pp. 19–41.

16. Sternheimer, R. M.; and Lihdenbaum, S. J.: Extension of the Isobaric Nucleon Model for Pion Production 1: Pion-Nucleon, Nucleon-Nucleon, and Anti-Nucleon-Nucleon Interactions. *Phys. Rev.,* vol. 123, no. 1, 1961, pp. 333–370.

Table 1. Isospins and Masses of Particles

Particle	Isospin	Isospin projection	Mass, GeV
p	1/2	1/2	0.9383
n	1/2	−1/2	.9396
π^+	1	1	.1396
π^o	1	0	.1350
π^-	1	−1	.1396
Δ^{++}	3/2	3/2	1.232
Δ^+	3/2	1/2	1.232
Δ^o	3/2	−1/2	1.232
Δ^-	3/2	−3/2	1.232
d	0	0	1.876

Table 2. Isospin Components of One-Pion Production Channels in
Nucleon-Nucleon (NN) Collisions

Channel	Isospin cross section $\sigma_{I_i I_p}$
$pp \to \pi^+ d$	σ_{10}^d
$pp \to pp\pi^o$	σ_{11}
$pp \to pn\pi^+$	$\sigma_{11} + \sigma_{10}$
$np \to \pi^o d$	$\frac{1}{2}\sigma_{10}^d$
$np \to np\pi^o$	$\frac{1}{2}(\sigma_{10} + \sigma_{01})$
$np \to nn\pi^+$	$\frac{1}{2}(\sigma_{11} + \sigma_{01})$
$np \to pp\pi^-$	$\frac{1}{2}(\sigma_{11} + \sigma_{01})$

Table 3. Isospin Cross-Section Parameters

[From ref. 9]

Parameter	σ_{10}^d	σ_{11}	σ_{10}	σ_{01}
α	6.030	3.772	15.28	146.3
β	1.700	1.262	0	0
m_o, MeV	1203	1188	1245	1472
Γ, MeV	134.3	99.02	137.4	26.49

Table 4. Meson Production Thresholds

Channel	Threshold, GeV
$NN \to NN\pi$	0.29
$NN \to NN\pi\pi$	.602
$NN \to NN\pi\pi\pi$	.934
$NN \to K\Lambda$	1.12
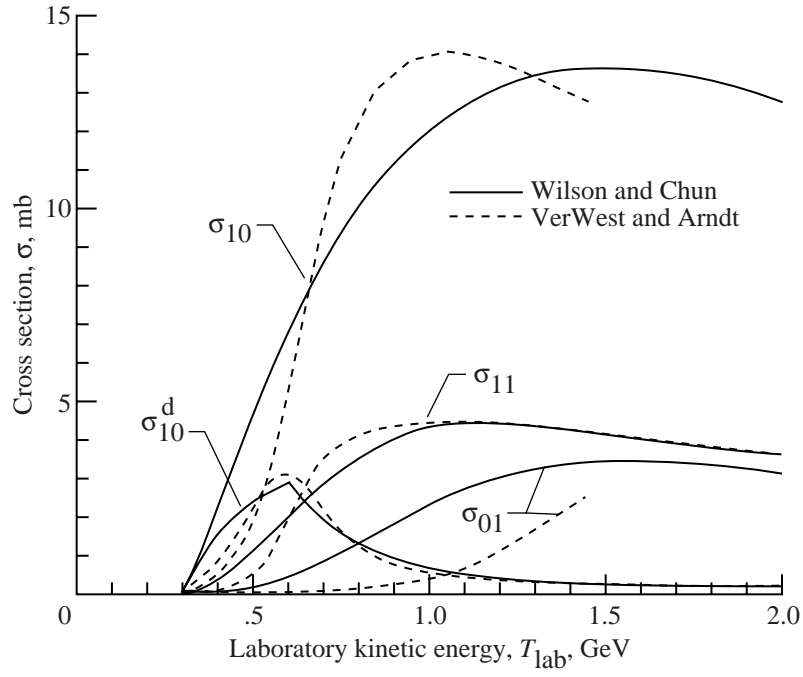
Figure 1. Comparison of isotopic cross sections σ for one-pion production versus laboratory kinetic energy of incident nucleon.
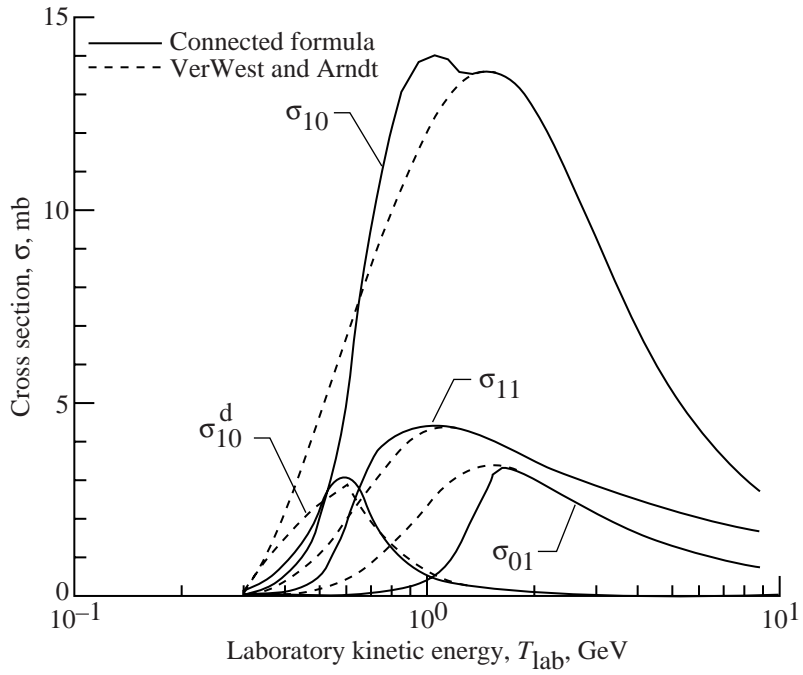
Figure 2. Comparison of isotopic cross sections σ for one-pion production versus laboratory kinetic energy of incident nucleon using connection of low- and high-energy fit equations.
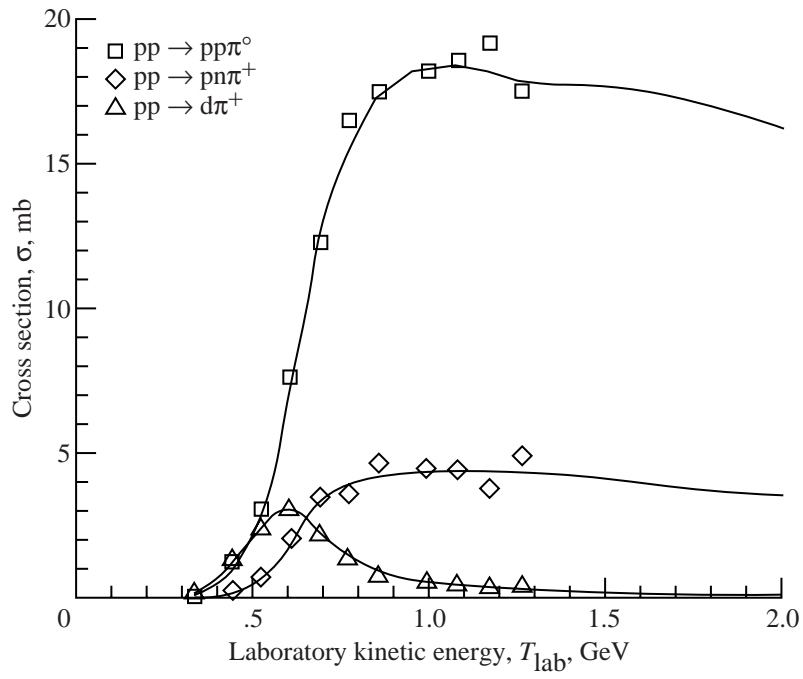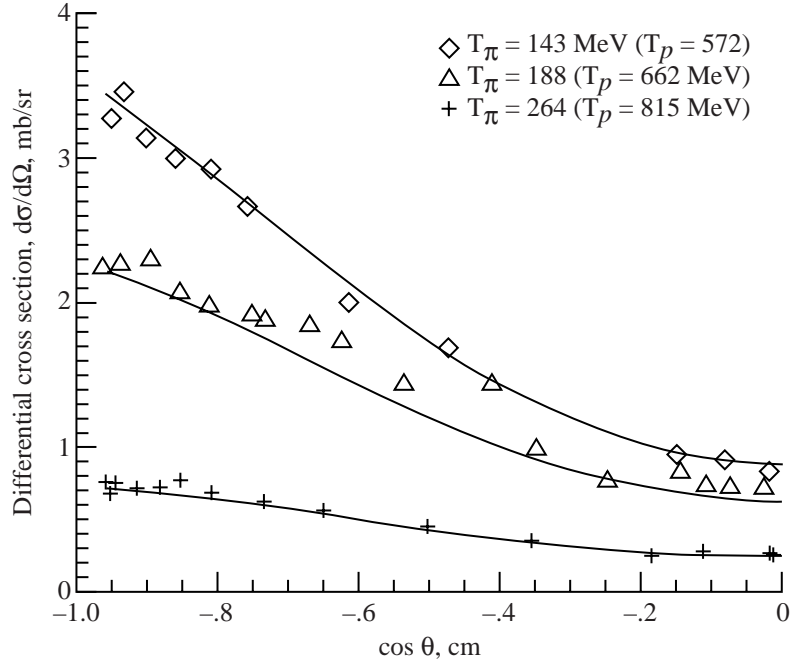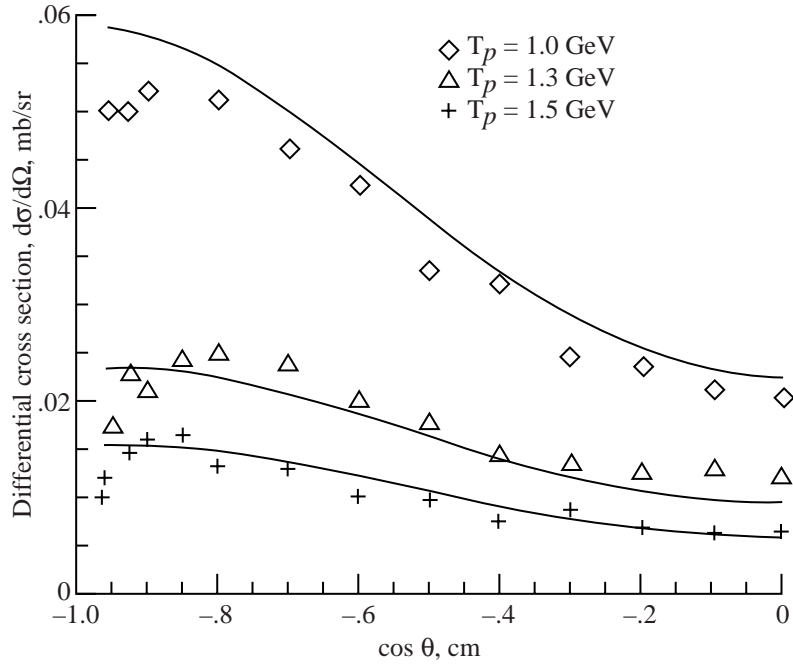
15

Figure 3. Comparisons of parametric fits to experimental data (ref. 10) for pion production in proton-proton (pp) collisions versus laboratory nucleon kinetic energy.

(a) $\pi^+ + d \rightarrow p + p$ reactions.

(b) $p + p \rightarrow \pi^+ + d$ reactions.

Figure 4. Comparisons of model fits to experimental data (ref. 11) for angular distributions; T_π is kinetic energy of pion; T_p is kinetic energy of proton.

17
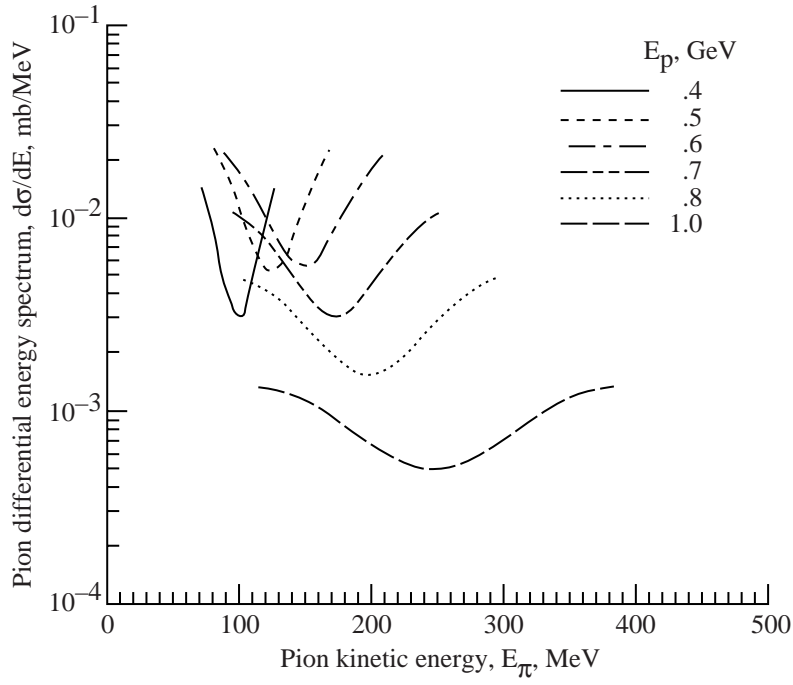
Figure 5. Secondary π^+ spectrum in $p + p \rightarrow \pi^+ + d$ reactions for several proton beams.
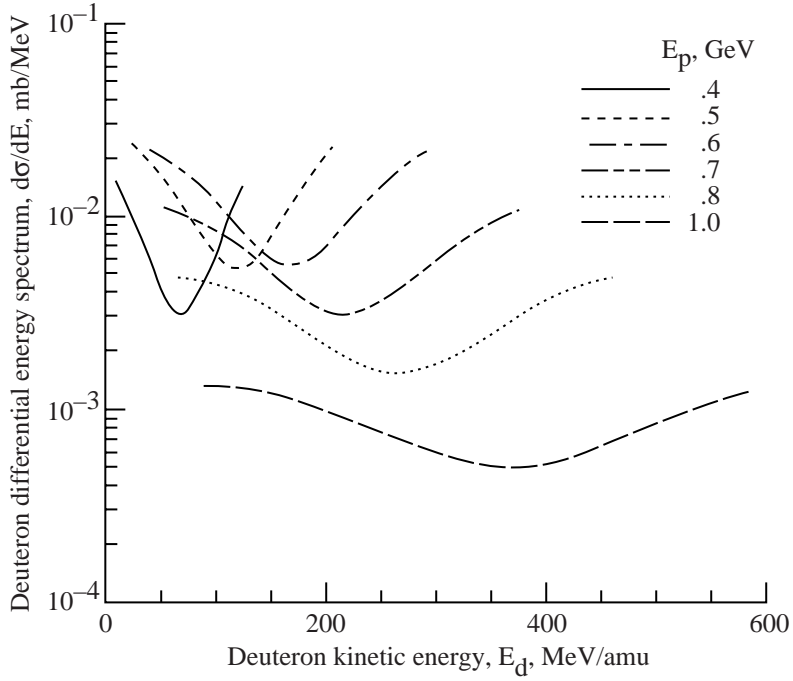
Figure 6. Secondary deuteron d spectrum in $p + p \rightarrow \pi^+ + d$ reactions for several proton beams.
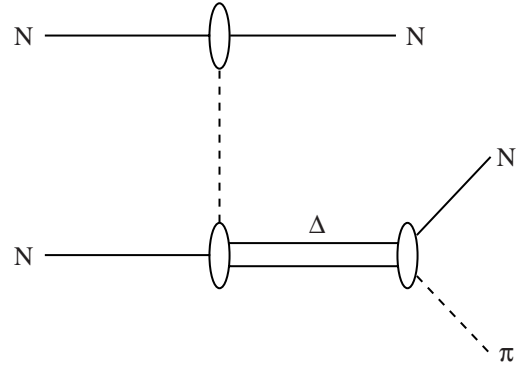
(a) NN → $N\Delta$ → NNπ.
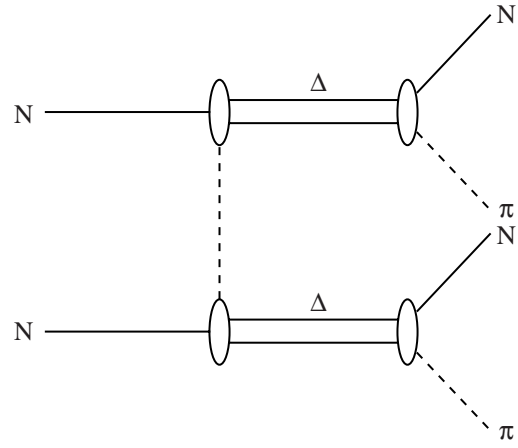
(b) NN → $\Delta\Delta$ → NN$\pi\pi$.
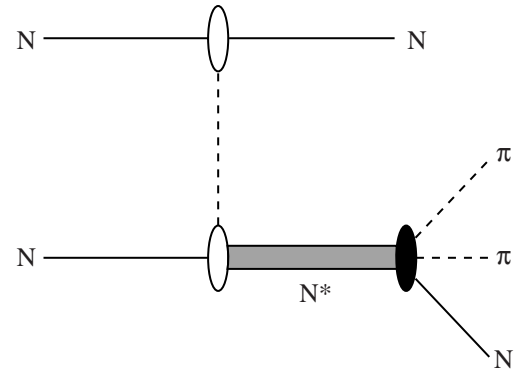
(c) NN → NN* → NN$\pi\pi$.

Figure 7. Diagrams for one-pion production through Δ or nucleon resonance N^* formation and decay in nucleon-nucleon (NN) collisions.
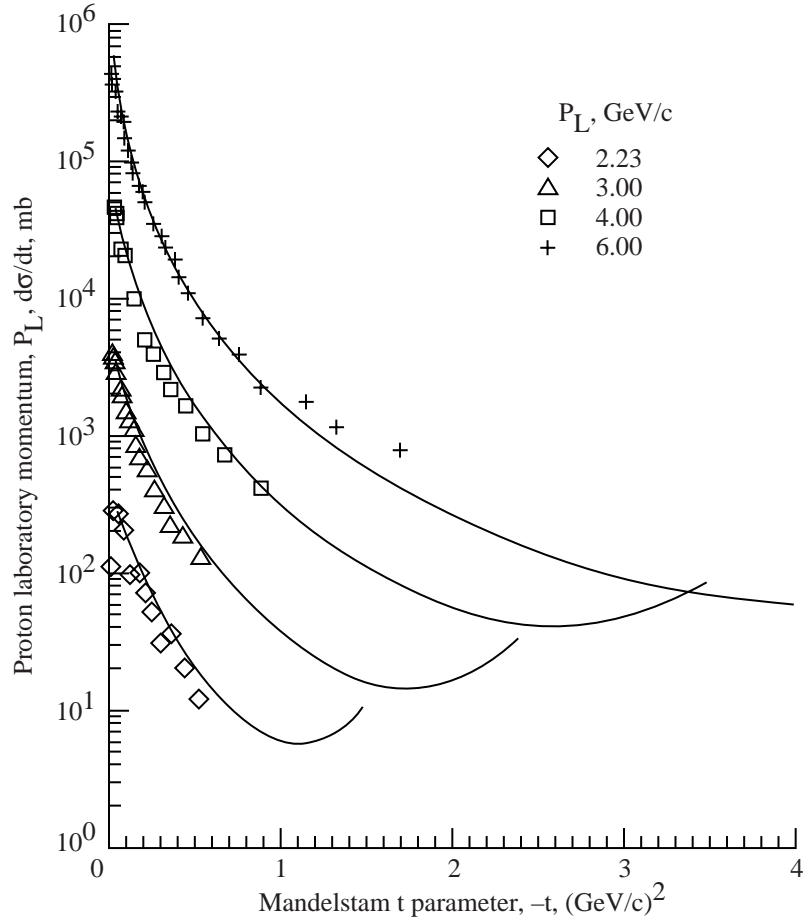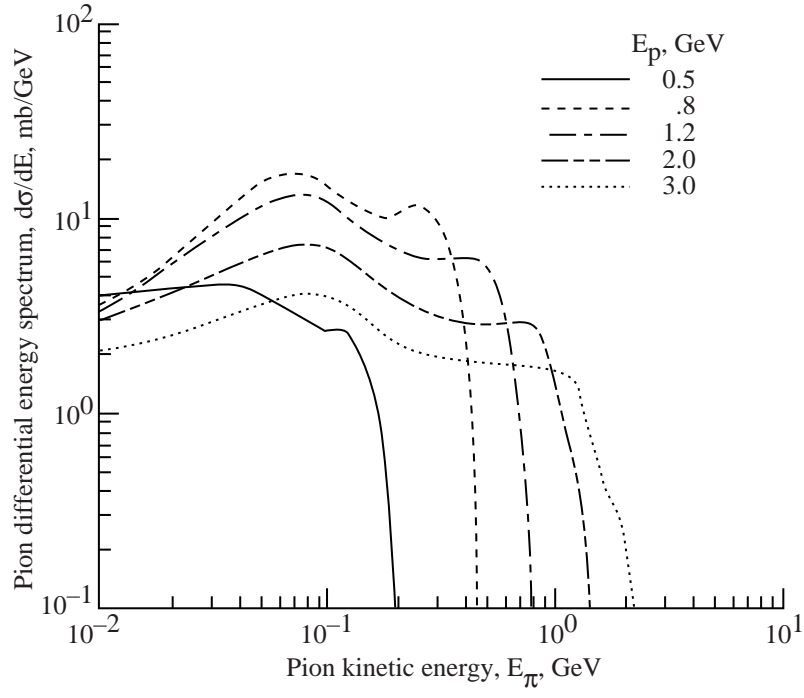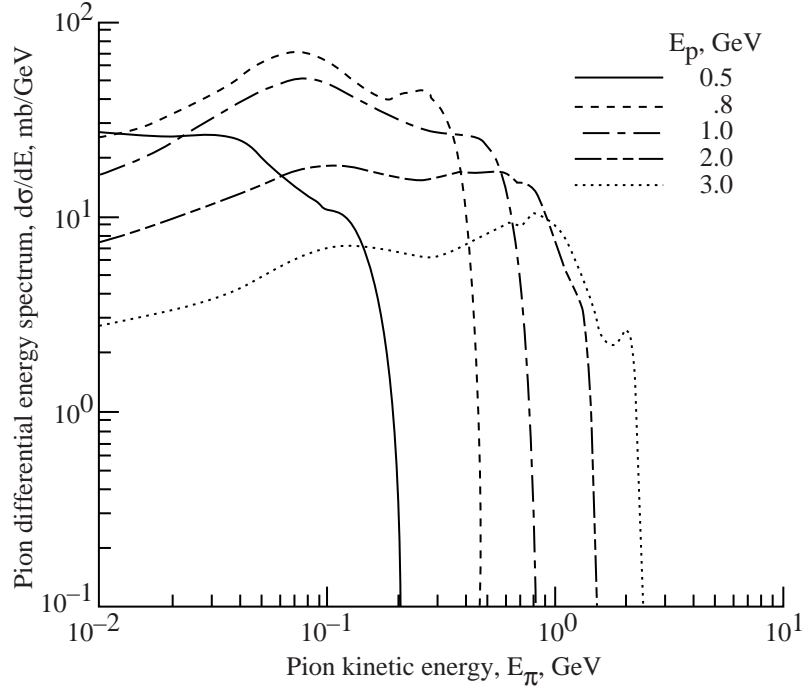
Figure 8. Comparison of model to experiments (refs. 10 and 15) for invariant distribution in $pp \to n\Delta^{++}$ reactions.

(a) $p + p \rightarrow \pi^+ + n + p$.

(b) $p + p \rightarrow \pi^0 + p + p$.

Figure 9. Calculations of pion (one-pion contribution) energy spectrum in proton-proton (pp) collisions at several beam energies.
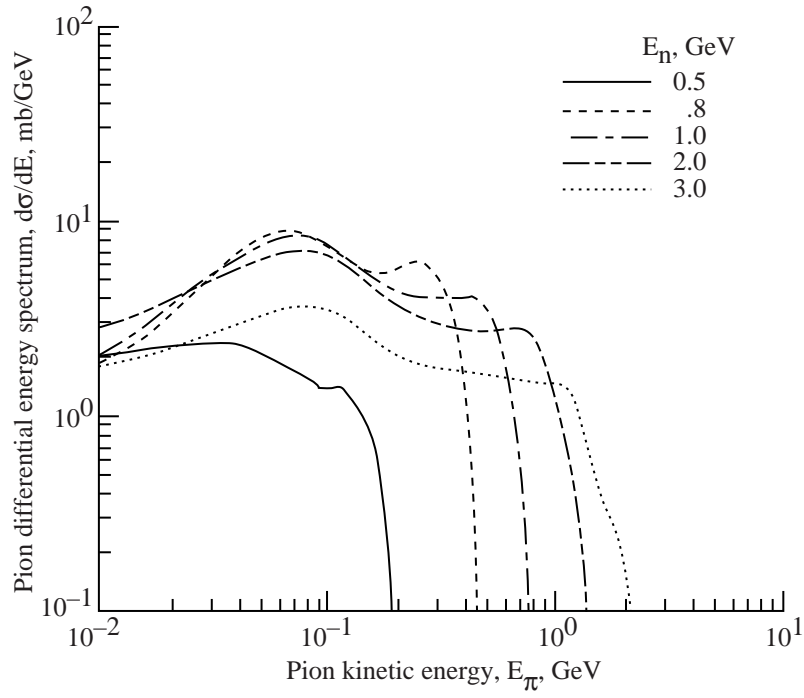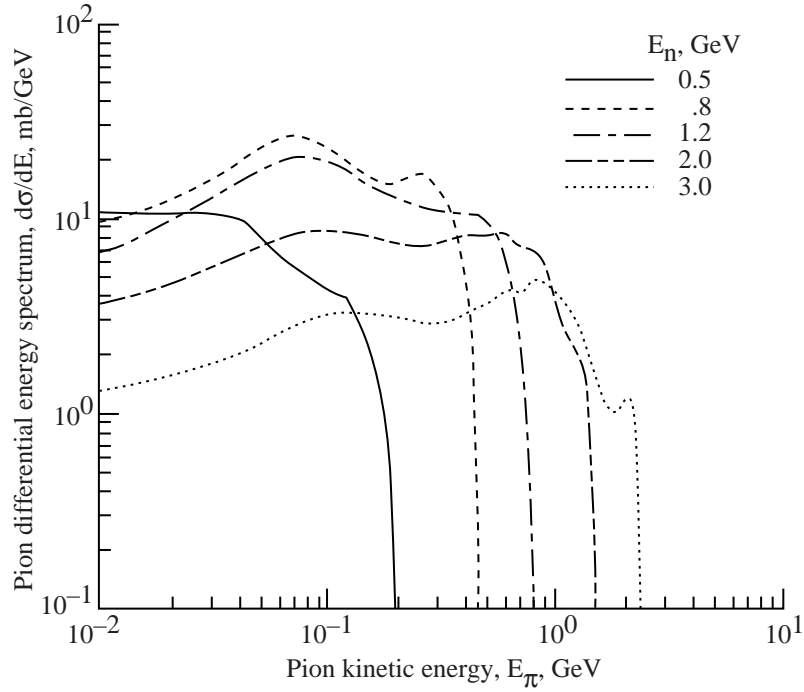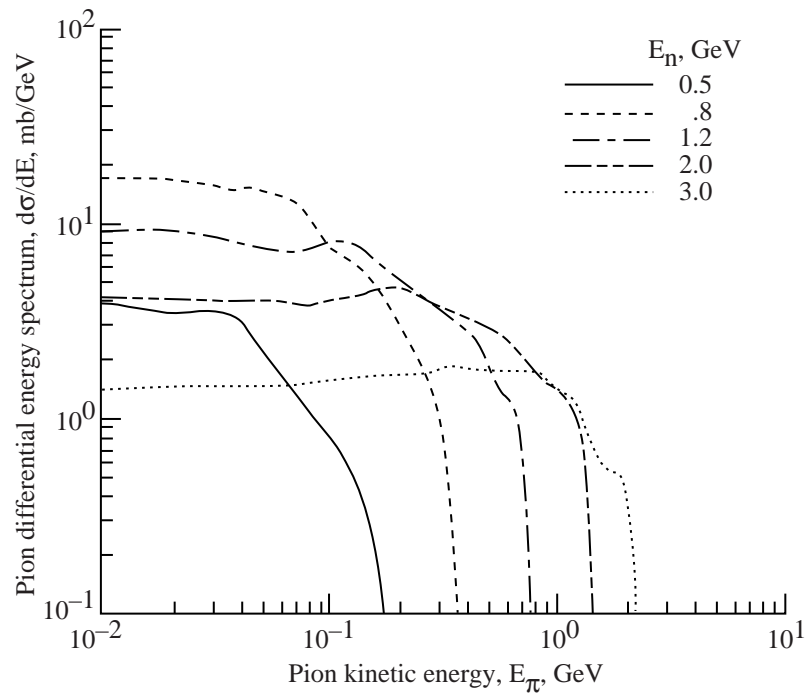
(a) $n + p \rightarrow \pi^+ + n + n$.

(b) $n + p \rightarrow \pi^0 + n + p$.

Figure 10. Calculations of pion (one-pion contribution) energy spectrum in neutron-proton (np) collisions at several beam energies.

(c) $n + p \rightarrow \pi^- + p + p$.

Figure 10. Concluded.

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 07704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE October 1998	3. REPORT TYPE AND DATES COVERED Technical Memorandum

4. TITLE AND SUBTITLE
Parameterizations of Pion Energy Spectrum in Nucleon-Nucleon Collisions

5. FUNDING NUMBERS
WU 199-45-16-11

6. AUTHOR(S)
Francis A. Cucinotta, John W. Wilson, and John W. Norbury

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

NASA Langley Research Center
Hampton, VA 23681-2199

8. PERFORMING ORGANIZATION REPORT NUMBER

L-17650

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA/TM-1998-208722

11. SUPPLEMENTARY NOTES
Cucinotta: Langley Research Center, Hampton, VA (now at Lyndon B. Johnson Space Center, Houston, TX); Wilson: Langley Research Center, Hampton, VA; Norbury: University of Wisconsin, Milwaukee, WI.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified–Unlimited
Subject Category 73   Distribution: Standard
Availability: NASA CASI (301) 621-0390

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
The effects of pion (π) production are expected to play an important role in radiation exposures in the upper atmosphere or on the Martian surface. Nuclear databases for describing pion production are developed for radiation transport codes to support these studies. We analyze the secondary energy spectrum of pions produced in nucleon-nucleon (NN) collisions in the relativistic one-pion exchange model. Parametric formulas of the isospin cross sections for one-pion production channels are discussed and are used to renormalize the model spectrum. Energy spectra for the deuteron related channels ($NN \rightarrow d\pi$) are also described.

14. SUBJECT TERMS
Space radiation; Atmospheric radiations; Pion production; Transport radiation

15. NUMBER OF PAGES
28

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102