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Wind Tunnel Data Response Surface Methods**

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Abstract

The integration of nonlinear neural network methods with conventional linear regression techniques is demonstrated for representative wind tunnel force balance data modeling. This work was motivated by a desire to formulate precision intervals for response surfaces produced by neural networks. Applications are demonstrated for representative wind tunnel data acquired at NASA Langley Research Center and the Arnold Engineering Development Center in Tullahoma, TN.

Introduction

Wind tunnel testing technology activities conducted and sponsored by NASA Langley Research Center since 1997 have focussed on the introduction of formal experiment design principles to empirical aerodynamics¹. These activities have served to bring to bear on aerodynamic research problems the powerful machinery of formal experiment design first introduced by R.A. Fisher and associates early in the 20th century² and used successfully since then in a wide range of industrial, scientific, and engineering applications. Collectively, these methods are described at NASA Langley as Modern Design of Experiments (MDOE), after a phrase from the literature of formal experiment design that distinguishes these methods from what is commonly called *classical* experiment design.

Classical experimentation methods have been popular for hundreds of years, and form the basis of conventional wind tunnel testing procedures in use today. The defining feature of classical testing

methods is an error control strategy that requires each independent variable to be changed one at a time, while holding all other variables constant. This method, formally described in the literature of experiment design as One Factor at a Time (OFAT) testing, typically involves changing the levels of the independent variable under study as a monotonically increasing function of time. This is the basis of the common *polar*, for example, which is a popular wind tunnel testing data structure that consists of a series of angle of attack levels set sequentially in a monotonically increasing sequence, with all other independent variables (Mach number, angle of sideslip, etc) held constant.

MDOE practitioners recognize certain weaknesses in the OFAT testing philosophy that can be overcome by formal experiment design methods that essentially defend against these shortcomings³. Contrary to assumptions inherent in OFAT testing, we never actually “hold everything else constant” when we change one variable at a time in a wind tunnel test. We simply make no intentional changes in the other independent variables. Changes do occur, nonetheless. For example, flow angularity fluctuations cause effective unscheduled changes in the angle of attack, blockage effects can cause an equivalent shift in Mach number, and cross-flow temperature gradients can induce assorted variations (especially in cryogenic high Reynolds number testing). Instrumentation and data systems drift over time, subtle differences can exist in procedures and techniques from one operator crew to another, etc, etc. Despite the best efforts to keep these factors constant, variations in individual data points are inevitable.

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A central weakness of OFAT testing is that it is single-point oriented, so the combined effects of all inevitable failures to “hold everything else constant” are directly reflected in the errors that exist in each individual data point. By contrast, MDOE methods are “data-ensemble” oriented. The data set is analyzed as an integrated whole, rather than on a single-point basis. The focus is on defining *relationships* between each response variable and the independent variables that cause it to change. If these relationships are well understood, it is possible to predict responses for any combination of independent variables in the range of those tested, not simply the specific points set in the test. Furthermore, such mathematical relationships (called response surface models) can be used to predict responses in other circumstances, e.g. in other tests or in flight. By focusing on the response surface as a whole rather than individual data points, and by adopting tactics intended to ensure that all errors are identically and independently distributed, the effects of local single-point errors can be made to substantially cancel. This results in higher precision than can be achieved on a single-point basis. Furthermore, the residual degrees of freedom – those points in excess of the minimum required to define the response surface – are available to assess the quality of predictions made by the model.

The most common method for defining response surface models in MDOE wind tunnel testing to date has been linear regression. Linear regression results in low-order polynomial functions of (possibly transformed) independent variables that can produce adequate predictions for a wide range of practical applications. This type of model is well suited for limited ranges of the independent variables, but in complex situations that can occur in certain aerodynamic testing this may not cover the entire range of interest. For response relationships rich in higher-order components, such as those featuring the near-discontinuous responses that are not uncommon in transonic aerodynamics, for example, extensions to conventional low-order linear regression will be needed to produce response surfaces adequate for reliable prediction. One promising possibility is the application of neural network methods to nonlinear response problems.

In the present paper, we have replaced the polynomial model by a Neural Network in constructing the Response Surface. The range of test design points could be enlarged to cover most test variables required range since the neural networks are capable to map a highly nonlinear hyper-surface. The identification of underlying model could also be

studied from the response surface. In addition to data interpolation performed by the neural networks, the analysis of derivative functions, variable sensitivity and interaction effects could be investigated even though we have not included in the present effort.

The widely used algorithms of neural networks in mapping function or constructions of response surfaces are the "Back Propagation" and "Radial Basis Function Networks". Since the model of neural networks is **nonlinear** in nature, the confidence and prediction interval analysis is not feasible by adapting the linear regression approach based on the statistical theorem. However, the importance of the confidence and prediction intervals of a constructed response surface is recognized for the MDOE application. Without the capability to estimate confidence or prediction intervals, the neural network is unable to provide the fitting goodness characteristics or the imperfection in the model. The confidence interval is the way to identify the systematical errors in the model or the model adequacy. Furthermore, the prediction confidence interval is able to estimate or forecast the uncertainty of response surfaces of the future observations. This is particular important to know the expected uncertainty since the data are not even available in a region of interest. Therefore, the effort of this work has been devoted to construct a special type of neural networks including the feature of linear regression to compute the response surface and its confidence and prediction intervals.

Radial Basis Function Network Application to Tunnel Data of the Alpha-Jet Model

Two neural network algorithms, which are capable of modeling response surfaces, have been selected to integrate with multiple linear regression to compute precision intervals. In the **first** approach, the Radial Basis Function Network (RBFN) is introduced to compute precision intervals. A brief description of Radial Basis Function Network and the integrated computation procedure of Confidence and Prediction intervals with the linear regression analysis are given in Appendix I. As an example, the application to modeling force data as a function of angles of attack for the TST Alpha Jet Model as shown in Figure 1 is included for data⁴ acquired from Tunnel 16T at the Arnold Engineering and Development Center (AEDC)

The force coefficients⁴ were taken at Mach Number 0.8 and Chord Reynolds Number 1.5 millions under transition-free configuration in the present application. The angle of attack ranges from -4 to 10

degree. The results obtained from the RBFN and the original data are plotted in Figure 2. The comparison of RBFN results and tunnel data is within the accuracy of tunnel measurement. The results of 95% Confidence Interval Half Width on the response surface from Eq. (1-4) are also shown in Figure 2 as the error band by the Coef-HI (or -UPPER) and Coef-LO (-LOWER). The results are satisfactory as expected. In Figure 3, the 95% CIHW (Confidence Interval Half Width) is and Coef-Residual, which is defined as

$$\text{Coef-Residual} = \text{Coef-RBFN} - \text{Coef-DATA}$$

are plotted for the purpose of comparison. The prediction interval was not computed in this example since we do not have sufficient data to reserve as the test file. The prediction interval formula will be applied in the second example.

Back Propagation Neural Network Application to Tunnel Data of the Alpha Jet Model.

In the **second** approach, the integration of back propagation neural networks and multiple linear regression has been constructed to compute response surface and the precision confidence interval.

Although the RBFN is capable to map multiple variable function with the rapid training process, the majority of function mapping has been carried out by Back Propagation (BP) Neural Networks. It is well known that the BP Neural Network has the powerful capability of function mapping to model the nonlinear response surface in a large numbers of parameters in a wide range. Thus the determination of the precision intervals for the BP nets is also necessary in the function mapping of neural networks application.

The typical back propagation network has an input layer, an output layer and one or more hidden layers. The network relationship is a non-linear function. The analysis of confidence and prediction intervals based on the statistical method is not available. The concept of integration of the linear regression method into the **last hidden layer** of the back propagation network (i.e. the hidden layer just before the output layer) is enable to map the response surface but also to evaluate the confidence and prediction intervals of the response surface. This is a special case of linear regression model as named in Ref. 5. The integration of the process is described as follows.

The first step is to train a selected design of a back-propagation neural network for the desired response surface with its inputs. After the network is satisfactorily trained, the linear regression will be incorporated in the trained BP nets. The processing elements of the last hidden in the nets will be treated as **regressor variables** in a multiple linear regression model. With the trained weights of the BP nets, each observation will provide the value of regressor variables, ϕ_i , and the response, F , known as the desired output for each input data set as

$$F = \sum_{i=1}^K w_i \phi_i + w_0 + \varepsilon$$

Where the ϕ_i 's are **regressor variables** functions of the input layer data and all weights in the trained Back Propagation (BP) Neural Network. The w_i 's are known as the **regression coefficients**. The error (or residual) of the regression is ε . The input variables in the cases of wind tunnel data include Mach number, Reynolds number, Angle of Attack and so on. The outputs, F , are force coefficients, e.g. lift, drag and pitching moment, for model force data.

In addition, a modified Back Propagation Net can be enhanced by introducing functions in terms of input variables linearly independent. The modified net, which is called as Functional-link nets⁶, is to enhance the original representation of input. The additional dimensions produced by these functions may be learned more readily in the hyperplanes. These functions typically consist of outer-product and functional enhanced modes of input variables. Some of the superior qualities of Functional-link nets have been demonstrated in the supervised train net by many examples in the literature. This technique of a simple representation of the net is illustrated in Figure 4. The Functional-link Back Propagation nets have been applied in the present work to map all force coefficients of a force model in the next section.

This integrated method applied to tunnel force data of the Alpha Jet Model with three typical variables, Mach number, Reynolds Number and Angle of attack, which consists of 25 test configurations conducted in the National Transonic Facility at NASA/Langley Research Center. Each test configurations ranges from 10 to 22 points of angles of attack. The total number of test points is over four hundred sets of data. The integrated method is able to map all force data into a **single** neural network. The confidence and prediction intervals associated with

this neural network are computed by Eq.(I-4) and Eq.(I-6) for the Alpha Jet Model data, respectively.

In the present investigation, the data ranges of Mach Number from 0.6 to 0.9 and Chord Reynolds number from 2.7 to 10 millions for transition-free configurations are available. The angle of attack ranges from -4 to 10 degree for most test conditions.

With half of all database for the training cases of the neural network, the remaining half set of data are reserved for the testing cases. The force and moment coefficients plots of a typical training case for Mach Numbers 0.8 are shown in Figure 5. It can be seen that the comparison of Neural Network-LSM prediction and tunnel data is good for those typical training flow conditions. The Coef-HI (-Upper) and Coef-LO (-Lower) are also plotted in Figure 5 for the range of the 95% confidence intervals. The 95% CIHF (confidence interval half-width) and Coef-Residual at tunnel condition M=0.8 are plotted in Figure 6. The results are satisfactory.

Concluding Remarks

Multiple linear regression methods have been integrated into two neural network algorithms, the Radial Basis Function Network and the Back Propagation Network. Both neural networks, which have nonlinear characteristics, are capable of constructing nonlinear response surfaces from the wide range of data sets obtained in typical MDOE applications. Response surface precision intervals (confidence and prediction intervals) are determined by linear regression analysis. Applications of the Radial Basis Function Network and Back Propagation integrated method to the force data sets of an Alpha Jet Model obtained from AEDC Tunnel 16T and NASA Langley NTF have shown satisfactory data mapping results. These results suggest that there is significant potential for neural networks to be applied in wind tunnel testing.

With their non-linear adaptive training and cross-validation capabilities integrated with formal, quantitative goodness-of-fit assessment techniques, it may be possible to largely automate the examination of complex, non-linear aerodynamic phenomena. MDOE methods can be used to select information-rich combinations of independent variables that are then used to train a neural network. Predictions by the network can be compared to an independent confirmation data set to determine if an adequate percentage of confirmation points lie within the precision interval of the response surface. If not,

additional training data can be acquired until adequate predictions are demonstrated throughout the inference space of interest. This has the potential of simultaneously defining prediction models that satisfy design precision requirements while ensuring that no more data are acquired than is necessary to do so.

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Appendix I

The Radial Basis Function Networks

The Radial Basis Function Network (RBFN) with N inputs and a scalar output, which is depicted in Figure I-1, can be expressed for a function approximation as

$$F(x_j, w_i) = \sum_{i=1}^k w_i \phi_i \left(\sum_{j=1}^N (x_j - c_{ij})^2 \right) + w_o \quad (I-1)$$

Where x_j 's are the inputs, ϕ_i 's are the given basis function and w_i 's are the weights. The Gaussian function is chosen as the Basis Function as shown in Figure I-1. The Gaussian Function with a radial-basis function argument that is used to form a network is called Radial Basis Function Networks (RBFN). The Gaussian function of the input variable x_j 's is of the form

Where

$$\phi_i(r_i^2) = e^{-\frac{r_i^2}{2\sigma_i^2}} \quad (I-2)$$

c_{ij} 's are the centers or mean values and σ_i 's are the standard deviations

$$r_i^2 = \sum_{j=1}^N (x_j - c_{ij})^2 \quad (I-3)$$

standard deviation of a normal distribution function of statistics.

By specifying a set of inputs, x_j 's and the corresponding desired output F, the values of the weights w_i 's can be determined using the linear **Least Squares Method** (LSM).

The above-described RBFN is a special case of Multiple Linear Regression models. The F is the desired output and is called as the Response. The ϕ_i is known as **regressors**, which are a specified function of inputs x_j 's.

The pattern unit (or regressors) in a RBF Network consists of center, c_{ij} and deviation, σ_i for each Gaussian function. A clustering algorithm is applied by Moody & Darken⁷ to determine the value of centers and a nearest neighbor heuristic to determine the deviation, σ_i . The **Linear Regression** or a gradient descent algorithm evaluates the weights of

the output function. The linear regression will be used in the present application. Therefore the confidence interval and predictive confidence interval can be determined by the available statistical method for this radial basis function network.

The description given herein has only a single (scalar) output for notational simplicity. There is no limitation of number of outputs. To extend the multiple outputs, another sets of weights should be introduced for additional desired outputs.

Implementation of RBFN and Computation of Precision Intervals

The RBFN algorithm is based on Moody and Darken work⁷. As the trained RBNF is accepted, the values of the regressors are determined for specified inputs and the target output. With these sets of inputs and corresponding outputs, the Precision Intervals for mean response can be obtained by the linear regression analysis. The Confidence Interval and Prediction Interval are able to compute that are based on the standard formulae given in the linear regression references (e.g. Ref. 8). The formulae are listed as follows:

A 100(1- α) percent half-width of **confidence interval (CIHW)** on the mean response at a particular point $\phi_{11}, \phi_{12}, \phi_{13}, \dots, \phi_{1k}$ is expressed as

$$t_{\alpha/2, n-p} \sigma \sqrt{x_1' (X'X)^{-1} x_1} \quad (I-4)$$

Where $t_{\alpha/2, n-p}$ is t-distribution quantiles, σ^2 is sum of squares residual/degree of freedom and

$$x_1' = [1, \phi_{11}, \phi_{12}, \dots, \phi_{1k}] \quad (I-5)$$

$$X' = [x_1, x_2, \dots, x_n]$$

A 100(1- α) percent half-width of **prediction interval (PIHW)** for the future observation is of the form

$$t_{\alpha/2, n-p} \sigma \sqrt{1 + x_1' (X'X)^{-1} x_1} \quad (I-6)$$

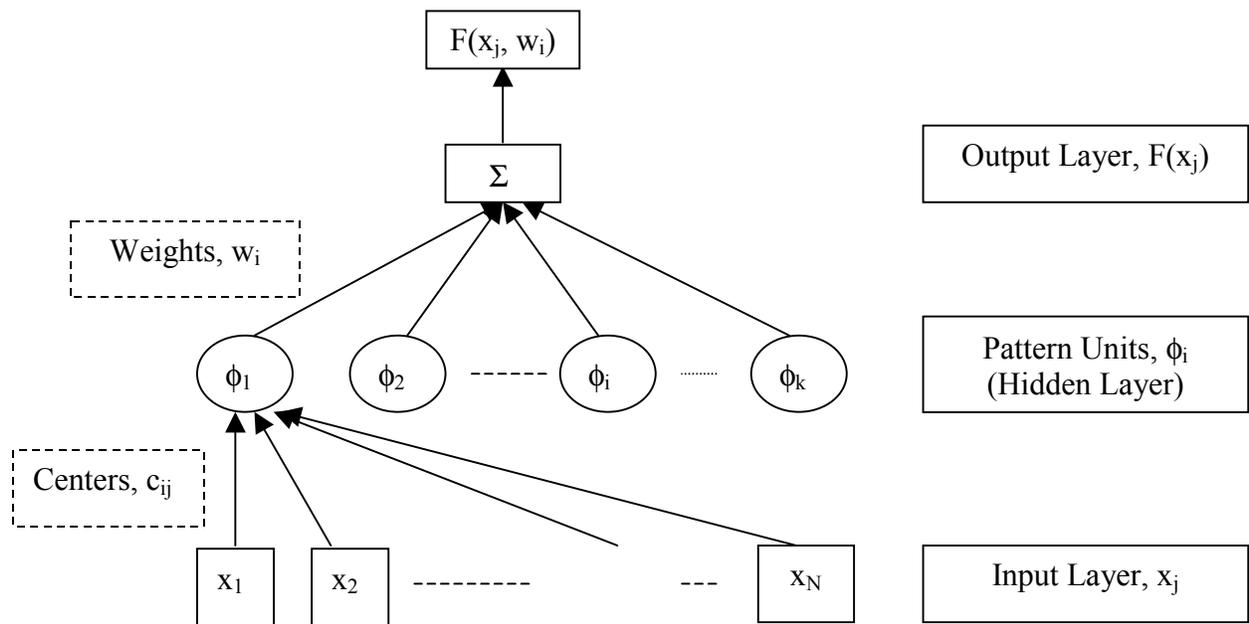
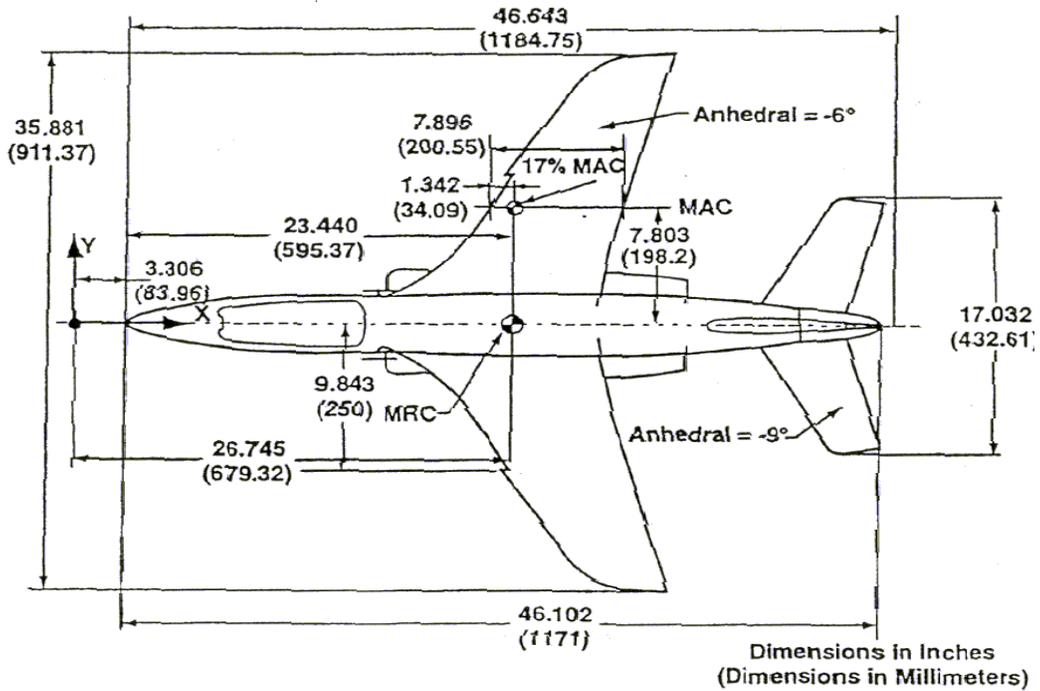


Figure I-1. Structure of a Radial Basis Function Network

Appendix U



TST model dimensions, top view.

Figure 1. Transonic Technology Wing (TST Alpha Jet) Model
From Ref 4.: Laster, M.L., Stanewsky, E, Sinclair, D.W. and Sickles, W.L.,
"Reynolds Number Scaling at Transonic Speeds," AIAA paper 98-2878, 1998

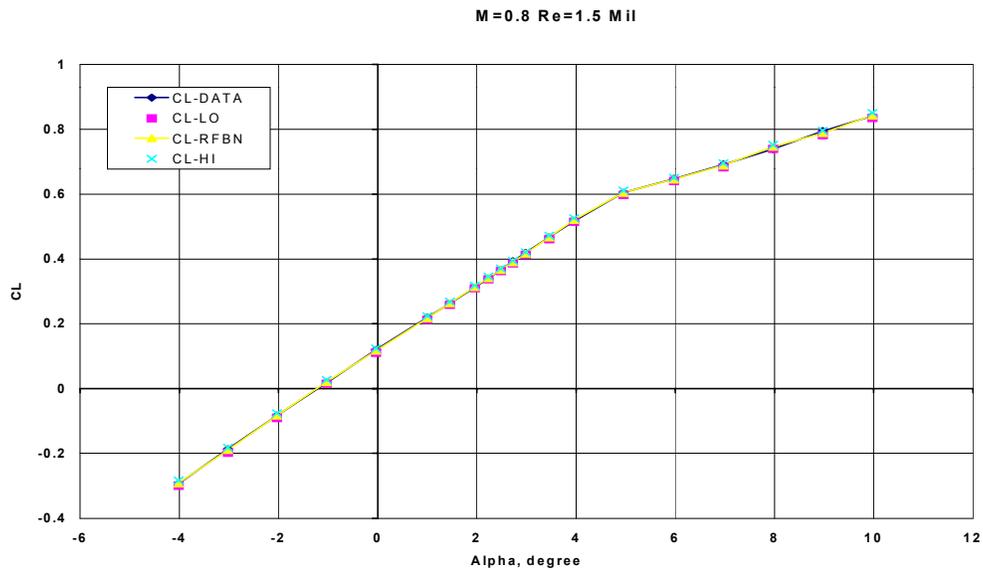


Figure 2(a) Lift Coefficient CL

M=0.8 Re=1.5 Mil

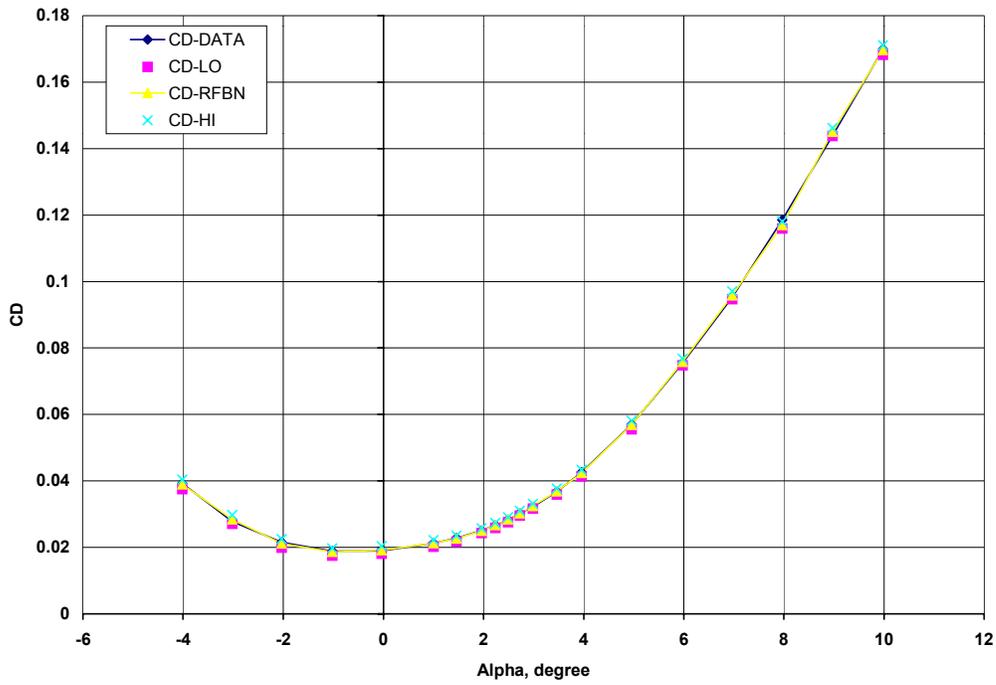


Figure 2(b). Drag Coefficient, CD

M=0.8 Re=1.5 Mil

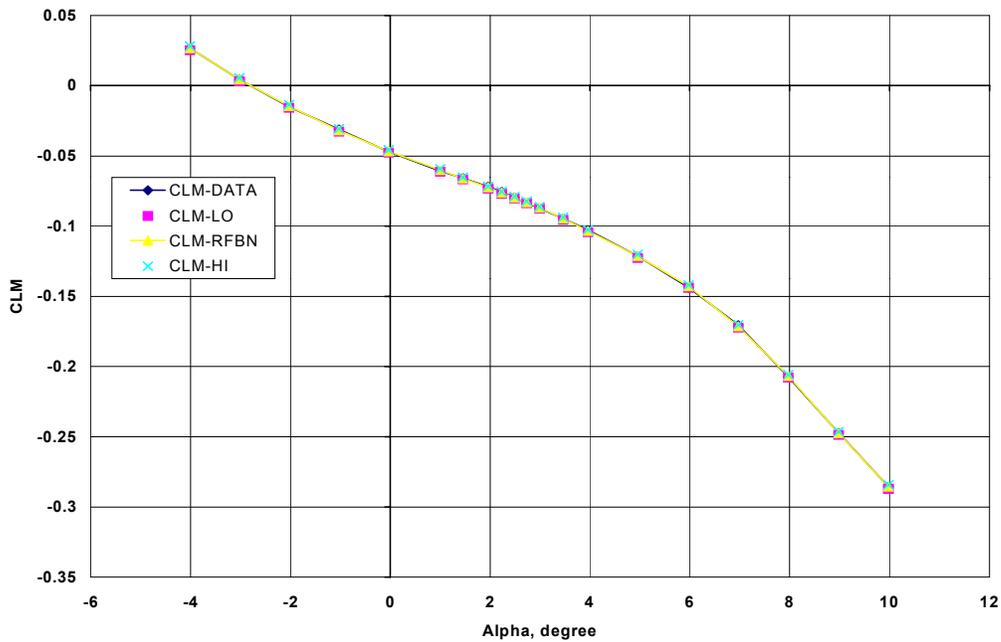


Figure 2(c). Pitching Moment Coefficient, CLM

Figure 2. Confidence Intervals with RBFN and DATA at M=0.8 and Re=1.5 Million.
Note: The Coef-HI and Coef-LO are indicated the range of the 95% Confidence intervals.

Lift Coefficient -- 95% Confidence Interval Half-Width
and CL-Residual, 16T Data

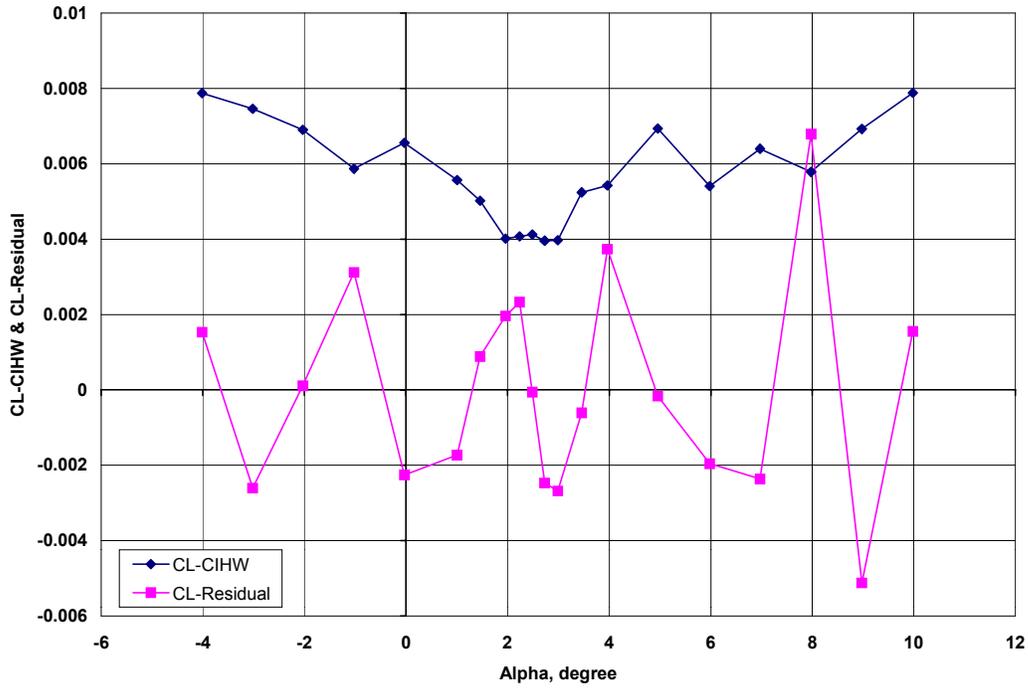


Figure 3(a). Lift Coefficient, $RMS(CL-Residual)=0.0027$

Drag Coefficient -- 95% Confidence Interval Half-Width
and CD-Residual, 16T Data

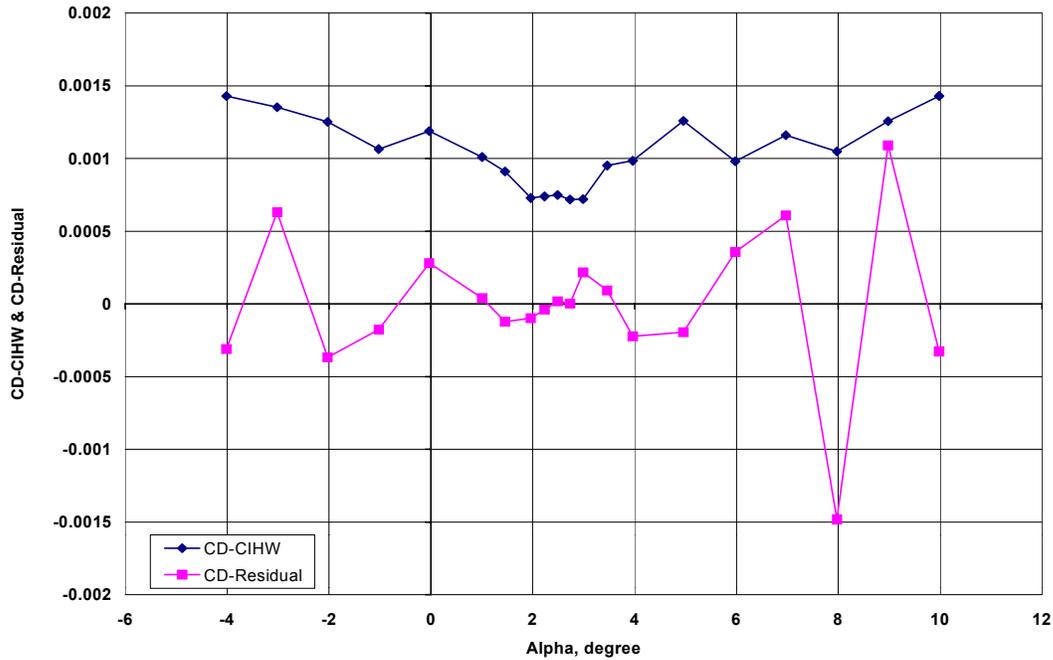


Figure 3(b). Drag Coefficient, $RMS(CD-Residual)=0.0005$

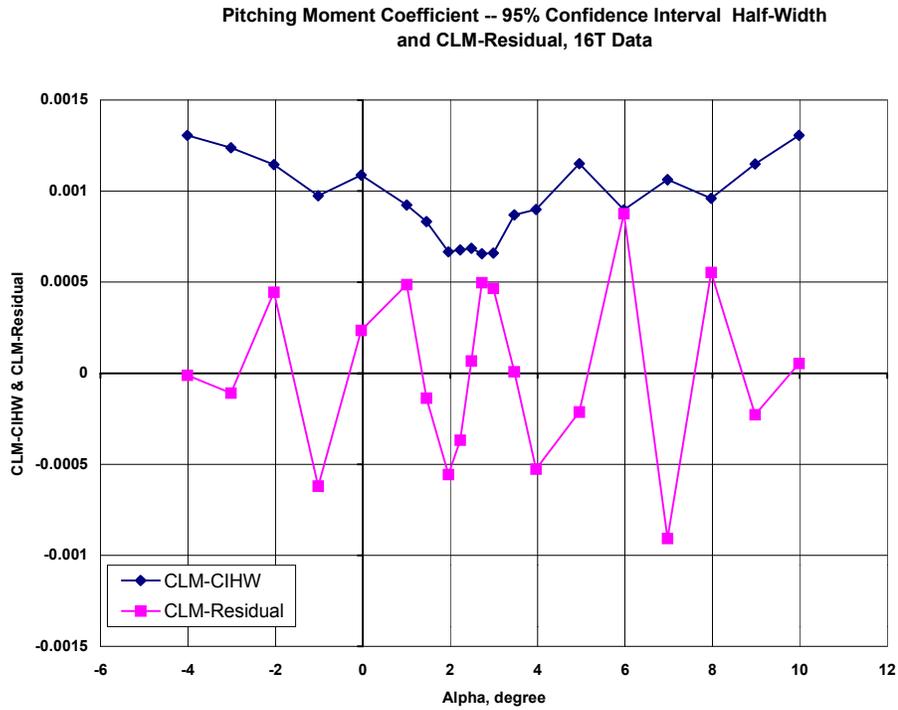


Figure 3(c). Pitching Moment Coefficient, RMS (CLM-Residual)=0.0005

Figure 3. The 95% Confidence Interval Half-Width and RBFN Coef-Residuals. Data Uncertainty for Tunnel 16T $\Delta CL=0.0048$, $\Delta CD=0.0009$ & $\Delta CLM=0.0025$.

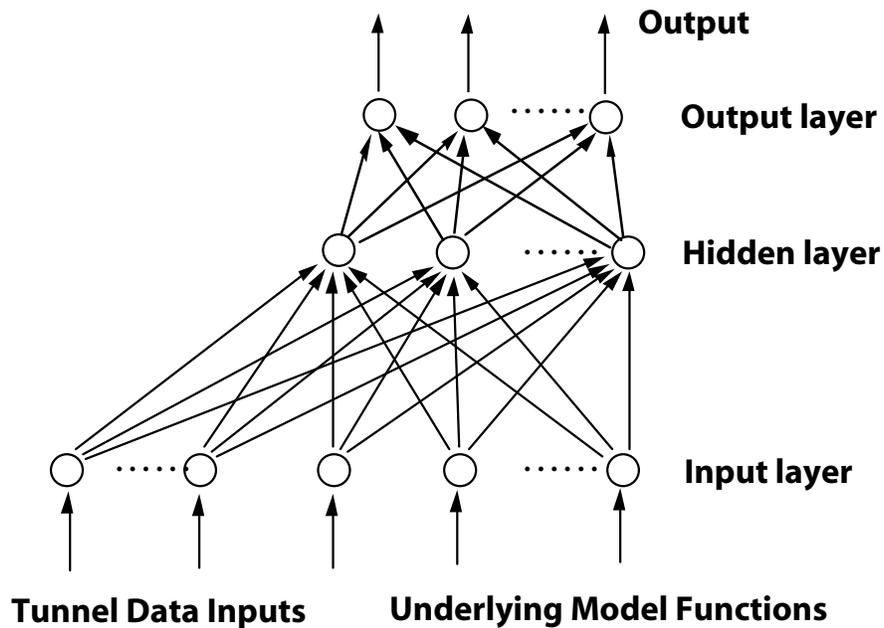


Figure 4. A Functional-Link Back Propagation Neural Network with Regular Tunnel Variable Data Inputs and Underlying Model Functions in terms of Regular Data as Additional Inputs.

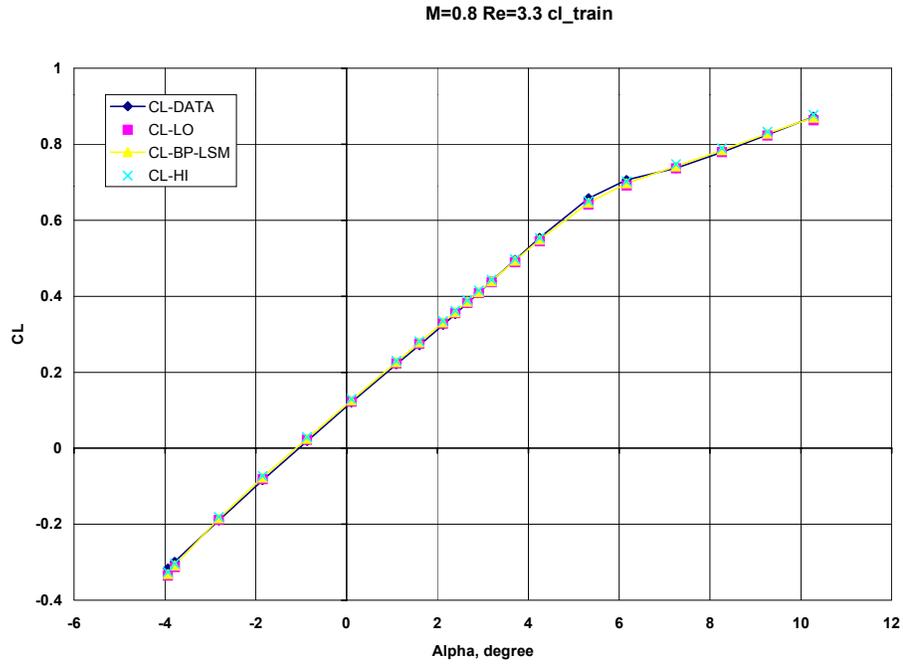


Figure 5(a). Lift Coefficient

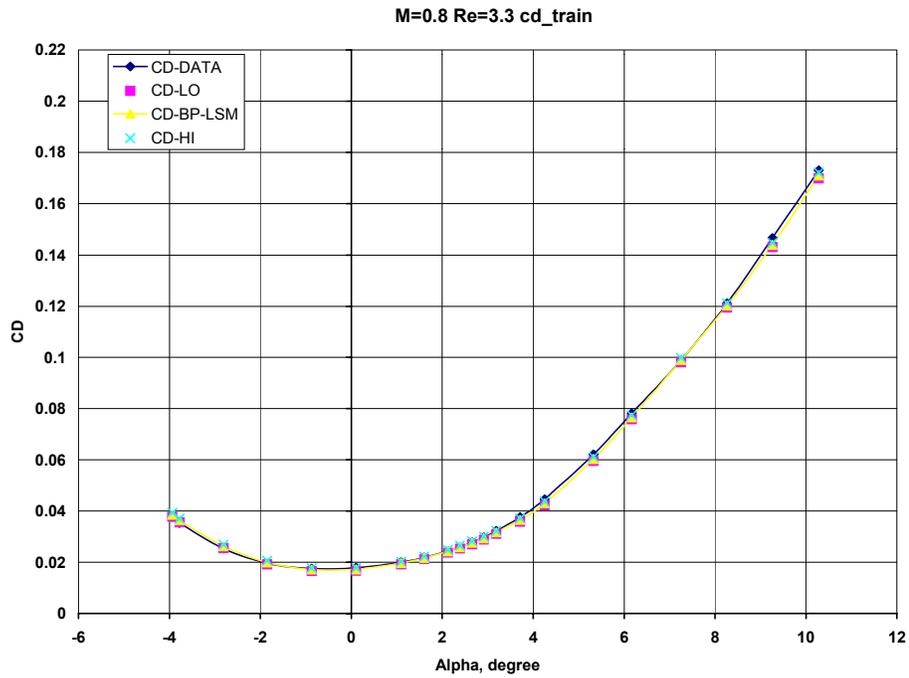


Figure 5(b). Drag Coefficient

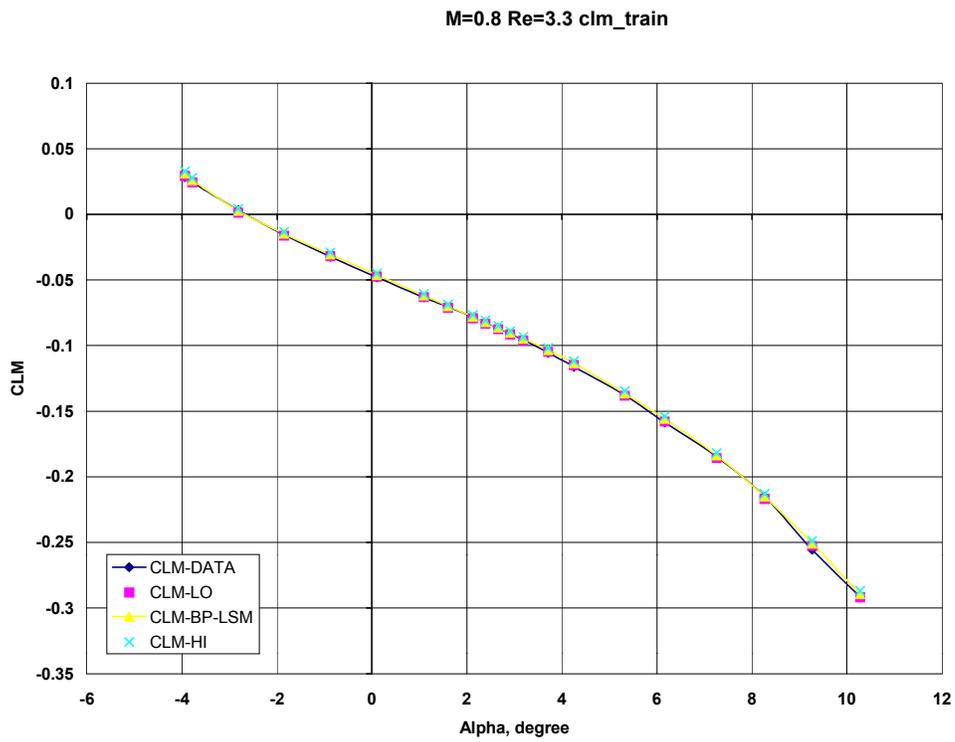
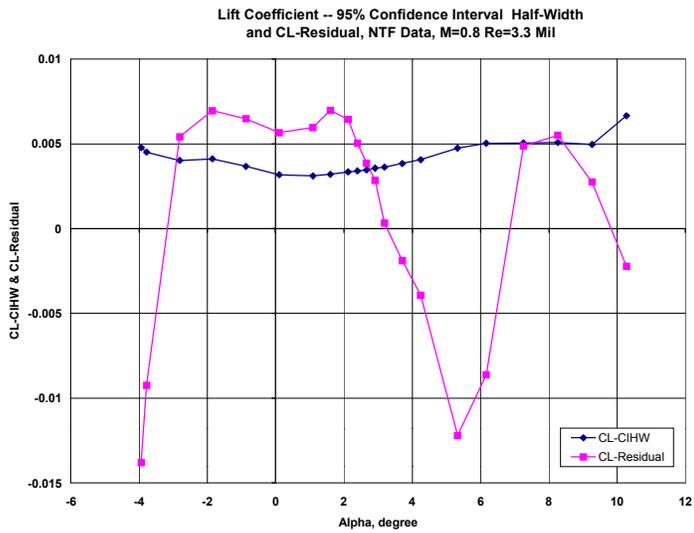


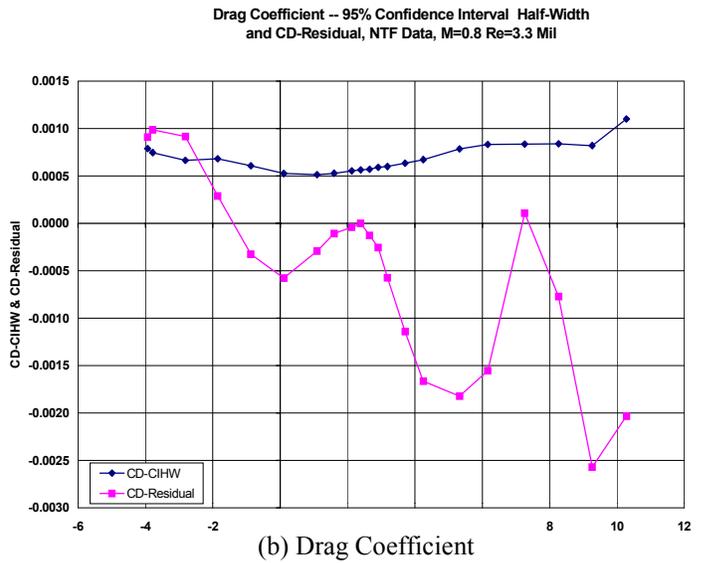
Figure 5 (c). Pitching Moment Coefficient

Figure 5. Training Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.8 Reynolds Number 3.3 mil for Force Coefficients.

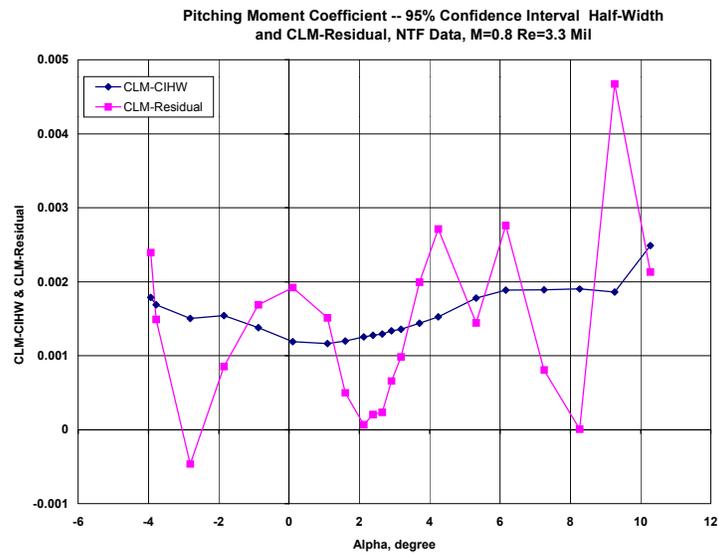
Note: Coef-LO and Coef-HI are 95% Confidence Intervals for Neural Networks-LSM 's Response Surface.



(a) Lift Coefficient



(b) Drag Coefficient



(c) Pitching Moment Coefficient

Figure 6. The 95% Confidence Interval Half-Width and BP-LSM Coef-Residuals at M=0.8 Re=3.3 Mil