Broadband Noise Predictions Based on a New Aeroacoustic Formulation

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Outline

• Introduction / Motivation
• Acoustic Formulation
• Model Problems
  Surface Pressure from Thin Airfoil Theory
  Velocity Scaling Properties, Directivity
• Broadband Noise Prediction
  Incident Turbulence Noise
  Comparison with Experiment
  (Paterson and Amiet, 1976)
Introduction / Motivation

• Broadband Noise Prediction Tools
  Airframe noise, ducted fan noise
  Incident turbulence, TE noise

• Time Domain Approach
  Acoustic Analogy
  Ffowcs Williams – Hawkings equation
  Decouples aerodynamics from acoustics
  Input from CFD or experiment
Acoustic Formulation

\[ \tilde{f} \geq 0 : \text{surface geometry} \]
\[ \hat{\nu} = \text{geodesic normal} \]
\[ \vec{x} = \text{observer position} \]
\[ \vec{y} = \text{source position} \]
\[ \vec{r} = \vec{x} - \vec{y} \]
\[ \cos \theta = \vec{r} \cdot \hat{e}_3 / r \]

The only restriction on the velocity vector is that it lie in the same plane as the surface.
Ffowcs Williams - Hawkings Equation

Loading Noise (Dipole)

\[
\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = - \nabla \cdot \left[ p \hat{n} H(\tilde{f}) \delta(x_3) \right] \\
= -p(x_1, x_2, t) H(\tilde{f}) \delta'(x_3)
\]

Solution:

\[
4\pi p'(\bar{x}, t) = - \int_{-\infty}^{t} \int_{\mathbb{R}^3} \frac{\delta(g)}{r} p(y_1, y_2, \tau) H(\tilde{f}) \delta'(y_3) d\bar{y} d\tau
\]

Free-space Green’s function: \( \frac{\delta(g)}{4\pi r}, \quad g = \tau - t + \frac{r}{c_0} \)
Formulation 1B

\[ 4\pi p'(\vec{x}, t) = \int_{\hat{f}>0} \left[ \frac{(\partial p/\partial \tau - V \partial p/\partial s) \cos \theta}{c_0 r (1 - M_r)} \right]_{\text{ret}} dS \]

\[ + \int_{\hat{f}>0} \left[ \frac{p \cos \theta}{r^2 (1 - M_r)} \right]_{\text{ret}} dS - \int_{\hat{f}=0} \left[ \frac{M_v p \cos \theta}{r (1 - M_r)} \right]_{\text{ret}} d\ell \]

- Given \( p(\vec{y}, t) \) on the surface, this formula yields the loading noise at an observer \( \vec{x} \) at time \( t \).

- For \( M \ll 1, r \gg \lambda \), the first integral dominates the signal.

- The contour integral vanishes at the trailing edge if the Kutta condition is satisfied.

- This formulation is valid for a rotating surface.
Formulation 1B

Test Cases

• Velocity Scaling Properties
• Directivity

Analytic Surface Pressure

• Thin Airfoil Theory (Amiet, 1975-6)
• Periodic Gust – Constant Frequency
• Flat Plate in Uniform Rectilinear Motion
Analytic Surface Pressure

Rectangular Surface at Constant Velocity

\[ 0 \leq x_1 \leq L_c, \quad -b \leq x_2 \leq b \]

\[ \mathbf{V} = [-U, 0, 0]^T \]

Upwash

\[ w(x_1, t) = w_0 e^{-ik(x_1-Ut)} \]

Surface pressure

\[ \Delta P(x_1, t) = \rho_0 U w_0 g(x_1, k) e^{ikUt} \]

\[ k = \omega / U = \text{constant}, \quad g(x_1, k) \text{ from thin airfoil theory} \]
Directivity

\[ L_c = 0.5 \text{ m} \]
\[ 2b = 2.0 \text{ m} \]
\[ M = 0.2 \]
\[ U = 68.6 \text{ m/s} \]
\[ w_0 = 0.05 \text{ } U \]

Test Cases

\[ \vec{x} : (r, \psi) \text{, } r = 3 \text{ m} \text{, } 0 \leq \psi \leq \pi \]
\[ f_1 = 1 \text{ kHz} \text{, } f_2 = 2 \text{ kHz} \]
Directivity

\[ f = 1 \text{ kHz} \]
\[ f = 2 \text{ kHz} \ (\text{scaled}) \]
Incident Turbulence Noise

Experiment: Paterson and Amiet (1976)

NACA 0012 Airfoil
- Chord = 0.23 m
- Span = 0.53 m
- $\alpha = 0$ degrees

Microphone:
- $r = 2.25$ m,
- $\theta = 90$ degrees

Screen

( nozzle )

( shear layer )

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2D Periodic Gust

**Upwash**

\[ w(x_1, x_2, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{w}(k_1, k_2) e^{-i[k_1(x_1-Ut)+k_2x_2]} \, dk_1 \, dk_2 \]

**Surface pressure**

\[ \Delta P(x_1, x_2, t) = \rho_0 U \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{w}(k_1, k_2) g(x_1, k_1, k_2) e^{i(k_1 Ut-k_2x_2)} \, dk_1 \, dk_2 \]

**Amiet (1975)**

Skewed gusts contribute little to the sound received by an observer in the plane \( x_2 = 0 \) (mid-span), i.e. \( k_2 = 0 \) is the only spanwise wave number to consider.
Broadband Surface Pressure

\[
\Delta P(x_1,t) \approx \rho_0 U \sum_{n=-N}^{N} A_{n,0} g(x, k_1,n,0) e^{i(k_1,n U t + \Phi_n)}
\]

\[
A_{n,0} = \left[ S_{ww}(k_1,n,0) \Delta k_1 \Delta k_2 \right]^{1/2}
\]

\[
S_{ww}(k_1, k_2) = \text{power spectral density of } w
\]

\[
\Phi_n = \text{random phase angle on } [0, 2\pi]
\]

Without explicit spanwise integration, this formulation can be expected to require amplitude scaling in the far field, but the spectral shape of the noise will be unaffected.
Turbulence PSD

2-Component von Karman Formula

\[
S_{ww}(k_1, k_2) = \frac{4}{9\pi} \frac{u^2}{k_e^2} \left[ 1 + \frac{k_1^2 + k_2^2}{\hat{k}_1^2 + \hat{k}_2^2} \right]^{7/3}
\]

\[
\hat{k}_i = \frac{k_i}{k_e}, \quad k_e = \frac{\sqrt{\pi}}{L_1} \frac{\Gamma(5/6)}{\Gamma(1/3)}
\]

\(L_1\) and \(u^2\) determined by experimental measurement
Broadband Prediction in Time

- Surface pressure input to Formulation 1B
- Five tunnel speeds: \( U = 40, 60, 90, 120, 165 \text{ m/s} \)
- Directivity and amplitude corrected for refraction through shear layer
- Acoustic pressure Fourier analyzed to convert to spectral density

**Predicted Signal**
\( U = 165 \text{ m/s} \)

\( p', \text{ Pa.} \)

\( t, \text{ seconds} \)
Far Field Noise Spectrum

Predictions

Experiment
- 40 m/s
- 60 m/s
- 90 m/s
- 120 m/s
- 165 m/s

Frequency, Hz

SPL, dB
Concluding Remarks

• A new solution to the FW-H equation provides a useful formulation for loading noise predictions.
• Although simple in form, Formulation 1B is applicable to rotating surfaces.
• A broadband noise prediction was shown to agree well with experimental measurement.
• The formulation’s simplicity lends itself well to statistical analysis of broadband noise.
• Future research will include the application of the statistical formulation to trailing edge noise.