TIME CORRELATIONS AND THE FREQUENCY
SPECTRUM OF SOUND RADIATED BY TURBULENT FLOWS

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ABSTRACT

Theories of turbulent time correlations are applied to compute frequency spectra of sound radiated by isotropic turbulence and by turbulent shear flows. The hypothesis that Eulerian time correlations are dominated by the sweeping action of the most energetic scales implies that the frequency spectrum of the sound radiated by isotropic turbulence scales as ω^4 for low frequencies and as $\omega^{-4/3}$ for high frequencies. The sweeping hypothesis is applied to an approximate theory of jet noise. The high frequency noise again scales as $\omega^{-4/3}$, but the low frequency spectrum scales as ω^2. In comparison, a classical theory of jet noise based on dimensional analysis gives ω^{-2} and ω^2 scaling for these frequency ranges. It is shown that the ω^{-2} scaling is obtained by simplifying the description of turbulent time correlations. An approximate theory of the effect of shear on turbulent time correlations is developed and applied to the frequency spectrum of sound radiated by shear turbulence. The predicted steepening of the shear dominated spectrum appears to be consistent with jet noise measurements.

I. Introduction

The nature of sound radiation by turbulent flow depends on properties of turbulent time correlations. For even if the energy contained in motions of any given spatial scale is known, say from the Kolmogorov theory, these motions will radiate sound at a frequency proportional to their inverse correlation time; this dependence evidently determines the frequency distribution of acoustic energy.

The present work applies theories of turbulent time correlations[1−5] to compute frequency spectra of sound radiated by isotropic turbulence and by shear turbulence. A question which must be answered at the outset is whether Eulerian or Lagrangian time correlations are required by this calculation. In the usual picture[6] of quadrupole sound sources, sound radiates from every point in an incompressible flow field because infinitesimal regions surrounding each point change shape without changing volume. In this picture, it is the Eulerian time correlations which are relevant. A calculation which treats moving fluid particles as acoustic sources must consider the random Doppler shift due to the motion.

Analyses of sound radiation by stationary turbulence have often assumed[7,8] that the space and time variables in the space-time correlation function are separable. Thus, for homogeneous turbulence,

$$< v_i(\mathbf{x}, t) v_i(\mathbf{x} + \mathbf{r}, s) > = F(\mathbf{r}) R(\tau/\tau^*) \tag{1}$$

where $\tau = t - s$ denotes time difference and τ^* denotes an integral time scale. But the temporal part of the space-time correlation function is scale dependent and Eq. (1) should be replaced by

$$< v_i(\mathbf{x}, t) v_i(\mathbf{x} + \mathbf{r}, s) > = F(\mathbf{r}) R(\mathbf{r}, \tau) \tag{2}$$

where

$$R(\mathbf{r}, 0) \equiv 1 \tag{3}$$

The goal of this paper is to assess the implications of using the more realistic model Eq. (2) in noise calculations. While it may be possible to compute single time quantities including the total acoustic power using a simplified formula such as Eq. (1), accurate prediction of two-time quantities like the acoustic power spectrum cannot be expected in general.

The specific form of Eq. (2) applied here is determined by the *sweeping hypothesis* for Eulerian time correlations,[1,3,4] which is described in more detail in Sect. II. Our previous work[9] applied this hypothesis to the calculation of the total acoustic power radiated by isotropic turbulence. In this idealized problem, the power spectrum is shown to scale as $\omega^{-4/3}$ for large frequencies and as ω^4 for small frequencies. Some numerical simulation data consistent with these scalings are described by Lilley.[10]

To demonstrate the importance of time correlations in predicting acoustic radiation from inhomogeneous flow, the classical theory of jet noise[11,12] is evaluated using Eqs. (2) and (3). This theory applies the Lighthill formula for far-field sound radiation, and assumes isotropic turbulence statistics. Although this theory is highly simplified, it provides a convenient way to compare the consequences of different theories of turbulent time correlations. The high frequency acoustic power is found to scale as $\omega^{-4/3}$, like isotropic turbulence, but the low frequency spectrum scales as ω^2. In contrast, dimensional arguments[12] lead instead to ω^2 scaling for low frequencies, and ω^{-2} scaling for high frequencies. It is shown that these scalings are recovered if the space-time correlation function is given the separated form postulated in Eq. (1). The difference in predicted scalings demonstrates the role of time correlations in acoustic analysis.

Although there is some experimental evidence that noise spectra measured at 90 degrees from the jet axis, for which this type of theory is most accurate, decay somewhat more slowly than ω^{-2} at high frequencies insofar as they exhibit power law scaling at all, these comparison with jet noise data are inconclusive. Quantitative predictions will depend on the numerical integration of the linearized Euler equations[13] with the present theory used to characterize the sound source.

Finally, we attempt to evaluate the power spectrum of sound radiated by shear turbulence. The concern of this analysis is not the so-called "shear noise,"[7] which is a global property of the mean flow, but the corrections to the acoustic frequency spectrum due to modifications by shear of the space-time correlation function. In contrast, the standard theory[7] does not consider these effects, but emphasizes instead the effects of shear on the directivity pattern of acoustic radiation.

Since time correlations in shear turbulence do not seem to have been investigated previously, they are computed using Leslie's perturbation theory[14] for the direct interaction

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approximation for shear turbulence.[15] As in Leslie's calculation, the acoustic spectrum is expressed as an expansion in powers of the mean velocity gradient. A consequence of the two-time properties of turbulence is that the radiated sound can depend on the antisymmetric part of the mean velocity gradient alone, unlike the Reynolds stress, for which such dependence is ruled out by general invariance arguments.[16] Thus, it may not be possible to characterize sound radiation by the Reynolds stress tensor. If the shear dominated noise is produced primarily by the near exit region of a jet, its frequency spectrum will scale as $\omega^{-10/3}$. This prediction compares reasonably well with a recent summary[17] of jet noise data.

II. General properties of acoustic frequency spectra

According to the Lighthill theory,[6] the frequency spectrum of sound radiated by homogeneous turbulence can be written as

$$p(\omega) = \Delta_{ijkl} I_{ijkl}(\omega) \tag{4}$$

In Eq. (4), Δ is Ribner's[7] geometric factor

$$\Delta_{ijkl} = \sum x_i x_j x_k x_l x^{-4}$$

where the vector x_i connects the source and observation point, and $\sum$ denotes symmetrization by summation or integration over source points symmetric with respect to the statistical symmetries of the turbulence. The dynamic factor I is

$$I_{ijkl}(\omega) = \frac{\omega^4}{16\pi^2 c^5} \int d\mathbf{p} \left[Q_{ik}(\mathbf{p},\omega) * Q_{jl}(-\mathbf{p},\omega) + Q_{il}(\mathbf{p},\omega) * Q_{jk}(-\mathbf{p},\omega) \right] \tag{5}$$

where $*$ denotes convolution with respect to ω and $Q_{ij}(\mathbf{p},\omega)$ is the two-time correlation function defined by

$$< v_i(\mathbf{p},\omega) v_j(\mathbf{p}',\omega') > = Q_{ij}(\mathbf{p},\omega) \delta(\omega + \omega') \delta(\mathbf{p} + \mathbf{p}')$$

Angular dependence of p in Eq. (4) arises from angular dependence of both Δ and Q_{ij}. Eqs. (4) and (5) are formulas for the acoustic power spectrum per unit mass of fluid; accordingly, the mass density ρ_0 of the fluid does not appear.

We recall the arguments which lead to Eqs. (4) and (5). The starting point is Lighthill's inhomogeneous wave equation. The solution of this equation is an integral containing space derivatives of time-retarded velocities. The dominant terms in the solution are identified as the space derivatives of the time-retarded quantities. These space derivatives are equal to time derivatives, which for time stationary conditions contribute the factor ω^4 in Eq. (5).

This argument leads to a fourth order velocity correlation; another approximation made in arriving at Eq. (5) is that the familiar quasinormal closure hypothesis is applied to close this correlation. An alternative to quasinormality which could potentially alter the sound pressure levels is considered in Ref. 18, but the corrections computed are very small. Another alternative which could alter the frequency scaling of aerodynamically generated sound is anomalous scaling of pressure correlations.[19] At this time, the occurrence of such anomalous scaling remains conjectural.

A simplified theory of sound radiation from shear flows results from a straightforward generalization[11] of Eq. (4) to inhomogeneous turbulence,

$$p(\omega) = \int \, d\mathbf{y} \; \Delta_{ijkl} I_{ijkl}(\omega, \mathbf{y}) \tag{6}$$

where $\mathbf{y}$ denotes position in flow field and the integration is over the flow region. In this case, Eq. (5) is generalized to

$$I_{ijkl}(\omega, \mathbf{y}) = \frac{\omega^4}{16\pi^2 c^5} \int \, d\mathbf{p}$$
$$[Q_{ik}(\mathbf{p}, \mathbf{y}, \omega) * Q_{jl}(-\mathbf{p}, \mathbf{y}, \omega) + Q_{il}(\mathbf{p}, \mathbf{y}, \omega) * Q_{jk}(-\mathbf{p}, \mathbf{y}, \omega)] \tag{7}$$

Since Eqs. (6) and (7) ignore flow-acoustic interaction,[13] their predictive capability is limited. These formulas will be used to compare the consequences of different theories of turbulent time correlations on noise spectrum predictions, not to obtain quantitative predictions.

III. Frequency spectrum of sound radiated by isotropic turbulence

For isotropic turbulence, Ribner's factor is

$$\Delta_{ijkl} = \delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj} \tag{8}$$

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The space-time correlation function of stationary, homogeneous, isotropic turbulence has the form

$$Q_{ij}^{(0)}(\mathbf{p}, \omega) = Q^{(0)}(p) P_{ij}(\mathbf{p}) \int_{-\infty}^{\infty} d\tau \, R^{(0)}(\mathbf{p}, \tau) e^{i\omega\tau} \tag{9}$$

where

$$P_{ij}(\mathbf{p}) = \delta_{ij} - p_i p_j p^{-2}$$

The superscript (0) is used in Eq. (9) and subsequently to identify isotropic quantities. The single time correlation function

$$4\pi k^2 Q^{(0)}(p) = E(p) \sim \begin{cases} \varepsilon^{2/3} p^{-5/3} & \text{for } p > k_0 \\ E_0(p) & \text{for } p < k_0 \end{cases} \tag{10}$$

exhibits Kolmogorov scaling for $p > k_0$; k_0 is the inverse integral scale which characterizes the beginning of the inertial range. The far infrared spectrum E_0 is included for completeness; however, it will not influence the frequency scaling of the acoustic power spectrum.

The time correlation function $R^{(0)}$ has an inertial range similarity form

$$R^{(0)}(p, \tau) = R^{(0)}(\tau\theta(p)) \tag{11}$$

where the functional form of $\theta(p)$ is determined by the temporal decorrelation mechanism. For the *sweeping hypothesis* for Eulerian time correlations,[1,3,4]

$$\theta(p) = C_D V p \tag{12}$$

where the sweeping velocity V in Eq. (12) is a property of the most energetic scales of motion. Thus, under the sweeping hypothesis, temporal decorrelation has a nonlocal character since it is determined for motions of any given scale by motions of possibly much larger scale. It is therefore possible that whatever contribution the most energetic scales may make to the total acoustic power, they always determine the frequency distribution of acoustic energy through their effect on the temporal decorrelation of the inertial range scales. In free shear flows, it is very likely that the sweeping velocity is a property of large-scale coherent structures. In this case, the sweeping velocity will not be predicted by turbulence models, which compute either inertial range or mean flow quantities.

Substituting Eqs. (8)-(12) in Eq. (5) yields

$$p(\omega) = \frac{\omega^4}{16\pi^2 c^5} \mathcal{G} \int_0^\infty 4\pi p^2 \, dp \, \frac{E(p)^2}{(4\pi p^2)^2} \int_{-\infty}^\infty d\tau \, R^{(0)}(\theta(p)\tau)^2 \, e^{i\omega\tau}$$
$$= \frac{\omega^4}{64\pi^3 c^5} \mathcal{G} \int_0^\infty dp \, \frac{E(p)^2}{p^2} \hat{R}^{(0)}(\omega/\theta(p))\theta(p)^{-1} \tag{13}$$

where the second equality defines $\hat{R}^{(0)}$ and $\mathcal{G}$ is the geometric factor

$$\mathcal{G} = \Delta_{ijkl} \oint dS(\mathbf{k}) \left[P_{ik}(\mathbf{k})P_{jl}(\mathbf{k}) + P_{il}(\mathbf{k})P_{jk}(\mathbf{k}) \right] = 64\pi$$

Since

$$\int_{-\infty}^\infty d\tau \, R^{(0)}(p,\tau)^2 = \mathcal{R} < \infty$$

the limit of Eq. (13) for small ω gives for the sweeping hypothesis,

$$p(\omega) = \frac{\mathcal{R}\omega^4}{\pi^2 c^5} \int_0^\infty dp \, \frac{E(p)^2}{p^2} \frac{1}{Vp}$$

This result expresses p as a functional of the exact energy spectrum $E(p)$. A simple approximation is obtained by substituting the cutoff Kolmogorov spectrum, Eq. (10) with $E_0 \equiv 0$:

$$p(\omega) = \frac{3}{16} \mathcal{R} \frac{C_K^2}{\pi^2} \frac{\omega^4}{c^5 V} \varepsilon^{4/3} k_0^{-16/3} \tag{14}$$

For large ω, introduce the change of variables $\xi = \theta(p)/\omega$ in Eq. (13) to obtain

$$p(\omega) = \frac{\omega^4}{\pi^2 c^5} \mathcal{R} \int_0^\infty \frac{\omega}{V} d\xi \, \frac{E(\omega\xi/V)^2}{(\omega\xi/V)^2} \hat{R}^{(0)}(1/\xi)$$
$$\sim \varepsilon^{4/3} V^{13/3} c^{-5} \omega^{-4/3} \tag{15}$$

where the Kolmogorov spectrum for $p > k_0$ from Eq. (10) is again substituted for $E(p)$. Summarizing the results of Eqs. (14) and (15),

$$p(\omega) \sim \frac{\rho\varepsilon}{c^5} \begin{cases} k_0^{-16/3} V^{-1} \varepsilon^{4/3} \omega^4 & \text{for } \omega \sim 0 \\ \varepsilon^{1/3} V^{13/3} \omega^{-4/3} & \text{for } \omega > Vk_0 \end{cases} \tag{16}$$

In this calculation, the acoustic power radiated by inertial range scales is determined through nonlocal sweeping effects by the sweeping velocity V, which is characteristic of the most energetic scales. The peak acoustic power occurs at a frequency $\omega \sim Vk_0$. Some preliminary evidence supporting the -4/3 scaling is given by Lilley[10].

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It is useful to compare the scalings of Eq. (16) with the scalings which would result from the hypothetical relation

$$\theta \sim p^{\alpha} \tag{17}$$

with $\alpha \neq 1$. An example is the *straining hypothesis* for Eulerian time correlations,[1,4] for which $\alpha = 2/3$. A simple calculation shows that for high frequencies

$$p(\omega) \sim \omega^{\beta}, \beta = 4 - (\frac{13}{3} + \alpha)\alpha^{-1} \tag{18}$$

In particular, for the straining hypothesis, $p \sim \omega^{-7/2}$. For any $\alpha < 1$, the high frequency spectrum predicted by Eq. (18) is steeper than the high frequency spectrum predicted by Eq. (16). This result can be understood in terms of the properties of time correlations: Eq. (12) shows that the correlation time of a given small scale, for which $p >> 1$ in suitable units, is shorter under the sweeping hypothesis than under Eq. (17) with $\alpha < 1$; therefore, the sweeping hypothesis predicts that any given small scale radiates sound at a higher frequency. This places more acoustic energy in higher frequencies and therefore makes the spectrum more shallow.

IV. Approximate theory of jet noise

In this section, the approximate scaling theory of jet noise[11,12] will be modified by introducing the time correlation function defined by Eqs. (11) and (12). In this theory, the sound radiated by an inhomogeneous flow is given by Eq. (7) in which all quantities are functions of position. The calculation will assume that the turbulence is locally isotropic. We also use the result[11] that the mean motion causes a Doppler shift of the acoustic spectrum as a whole, but does not alter the frequency scaling of the spectrum.

The theory uses similarity solutions to describe the flow field and suppresses all variation in cross-stream planes by averaging. Thus, all flow variables $f(x, y, z)$ are replaced by average values

$$\bar{f}(y) = \frac{1}{A(y)} \int dx dz \ f(x, y, z)$$

where $A(y)$ denotes the jet area at downstream distance y. The theory further assumes that there are two similarity regimes for a jet: the exit and the fully developed region. To simplify the notation, these regions will be taken to extend over $0 \leq y \leq L$ and $L \leq y < \infty$ respectively where y is the axial coordinate; the transition region between these similarity

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regimes will be ignored. L can be assumed to equal about $6.0D$ where D denotes the exit diameter of the jet.

A. *The fully developed region*

The classical similarity assumption for a fully developed jet is that the total momentum flux

$$\mathcal{M} = \rho \int dx dz \ U(x, y, z)^2$$

is constant downstream and that all cross-sectional averages scale with $\mathcal{M}$ and y only. In terms of the exit velocity U and diameter D,

$$\mathcal{M} \sim U^2 D^2 \tag{19}$$

In particular, this hypothesis gives the well-known scaling for average velocity

$$\bar{U}(y) \sim \mathcal{M}^{1/2} y^{-1}$$

and also determines the downstream variation of cross-section averages of turbulence quantities; thus,

$$\bar{\varepsilon}(y) \sim \mathcal{M}^{3/2} y^{-4} \tag{20}$$

It follows from Eq. (20) that the cross-section averaged spectrum satisfies

$$\bar{Q}^{(0)}(k, y) \sim \mathcal{M} y^{-8/3} k^{-11/3} \tag{21}$$

In Eqs. (20) and (21), $\sim$ denotes equality up to a universal constant; all dimensional parameters appear on the right hand side. In principle, these constants can be found from measurements.

Denote by $k_0(y)$ the integral scale at downstream location y. Then evidently, $k_0(y) = \sigma/y$ where σ is a universal constant proportional to the inverse spread rate of the jet. The total energy is found by integrating Eq. (21) over all $k \geq \sigma/y$ and leads to

$$\bar{K}(y) \sim \mathcal{M} y^{-2}$$

Introduce following Lilley[11]

$$p(y, \omega) = \int dx dz \ p(\omega, x, y, z) = A(y) \bar{p}(y, \omega)$$

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Under the sweeping hypothesis, the cross-section average of the frequency convolution in Eq. (7) has the similarity form

$$\frac{1}{A(y)} \int dx dz \ R^{(0)}(\omega/Vp) * R^{(0)}(\omega/Vp) = \frac{1}{V(y)p} F(\frac{\omega y}{\mathcal{M}^{1/2}p}) \sim \frac{y}{\mathcal{M}^{1/2}p} F(\frac{\omega y}{\mathcal{M}^{1/2}p}) \quad (22)$$

The subsitution of either an eddy turnover time like $\bar{K}/\bar{\varepsilon}$ or the inverse mean strain rate for the sweeping velocity will alter the proportionality constant in Eq. (22). It is likely that use of either of these time scales in calculations will require adjustment of this constant for each application.

Now substitute the results of Eqs. (21) and (22) in Eq. (7), then integrate over wavenumber p from σ/y to ∞. The result of this calculation is

$$p(y, \omega) \sim c^{-5} \omega^4 \mathcal{M}^{3/2} y^{-7/3} \int_{\sigma/y}^{\infty} dp \ p^{-19/3} F(\frac{\omega y}{\mathcal{M}^{1/2}p})$$

$$= c^{-5} \omega^{-4/3} \mathcal{M}^{25/6} y^{-23/3} G(\frac{\omega y^2}{\mathcal{M}^{1/2}}) \quad (23)$$

In Eq. (23), G is defined by

$$G(\frac{\omega y^2}{\mathcal{M}^{1/2}}) = \int_0^{\omega y^2 \mathcal{M}^{-1/2} \sigma^{-1}} d\zeta \ \zeta^{13/3} F(\zeta)$$

Note that by integrating over ω, we obtain

$$\int_{-\infty}^{\infty} d\omega \ p(y, \omega) \sim c^{-5} \mathcal{M}^{25/6} y^{-23/3} (y^2 \mathcal{M}^{-1/2})^{-1/3} \sim c^{-5} \mathcal{M}^4 y^{-7}$$

which is the y^{-7} law for power radiated by each cross-section.[12]

For small arguments,

$$G(\omega y^2 \mathcal{M}^{-1/2}) \sim (\omega y^2 \mathcal{M}^{-1/2})^{16/3} \quad (24)$$

For large arguments, if F decays exponentially or faster at infinity, then

$$G(\omega y^2 \mathcal{M}^{-1/2}) \sim \text{constant} \quad (25)$$

The power spectrum $p(\omega)$ is evaluated by integrating Eq. (23) with respect to y; since the integral converges at $y = \infty$, it can be extended to ∞ for analytical convenience, and the result is

$$p(\omega) \sim c^{-5} \omega^{-4/3} \mathcal{M}^{25/6} \int_L^{\infty} dy \ y^{-23/3} G(\frac{\omega y^2}{\mathcal{M}^{1/2}}) \quad (26)$$

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In view of Eq. (25), this integral converges in the limit $\omega \to \infty$; therefore, in this limit

$$p(\omega) \sim \omega^{-4/3} \mathcal{M}^{25/6} L^{-20/3} \tag{27}$$

To evaluate the limit of Eq. (26) when ω is small, it is convenient to introduce the change of variable $\zeta^2 = \omega y^2 \mathcal{M}^{-1/2}$. The result is

$$p(\omega) \sim c^{-5} \omega^2 \mathcal{M}^{5/2} \int_{\omega^{1/2} L \mathcal{M}^{-1/4}}^{\infty} d\zeta \; G(\zeta^2) \zeta^{-23/3}$$

In view of Eq. (24), $G(\zeta^2) \sim \zeta^{32/3}$ for small ζ, therefore, this integral converges for small arguments and when $\omega \to 0$,

$$p(\omega) \sim \omega^2 \mathcal{M}^{5/2} \tag{28}$$

Summarizing Eqs. (27) and (28)

$$p(\omega) \sim c^{-5} \begin{cases} U^5 D^5 \omega^2 & \omega \sim 0 \\ U^{25/3} D^{5/3} \omega^{-4/3} & \omega \sim \infty \end{cases} \tag{29}$$

where Eq. (19) has been substituted in Eqs. (27) and (28) and the result $L \sim D$ is used to eliminate L.

B. *The exit region*

The exit region of a jet has the scaling properties of an axisymmetric mixing layer: there is a constant velocity scale, and the area of the mixing region increases linearly downstream. Thus,

$$\bar{U}(y) \sim y^0 \tag{30}$$

$$A(y) = \int dx dz \; \sim yD \tag{31}$$

Then the scalings of Eqs. (20) and (21) are replaced by

$$\bar{\varepsilon} \sim U^3 y^{-1} \tag{32}$$

$$\bar{Q}^{(0)}(p, y) \sim U^2 y^{-2/3} p^{-11/3} \tag{33}$$

and

$$\frac{1}{A(y)} \int dx dz \; R^{(0)}(\omega/Vp) * R^{(0)}(\omega/Vp) \sim \frac{1}{Up} F(\frac{\omega}{Up}) \tag{34}$$

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Substituting Eqs. (30)-(34) in Eq. (7),

$$p(y,\omega) \sim \omega^{-4/3} c^{-5} y^{-1/3} U^{25/3} D H(\omega y U^{-1}) \qquad (35)$$

where

$$H(\omega y/U) = \int_0^{\omega y/U} d\xi \ \xi^{13/3} F(\xi) \qquad (36)$$

Integrating Eq. (35) over ω leads to

$$\int d\omega \ p(y,\omega) \sim y^{-1/3} U^{25/3} D(yU^{-1})^{1/3} \sim U^8 D$$

which is consistent with the y^0 law for power contributed by each downstream section.[12] Integrating Eq. (35) with respect to y,

$$p(\omega) \sim D\omega^{-2} U^9 c^{-5} \int_0^{\omega L/U} d\zeta \ \zeta^{-1/3} H(\zeta) \qquad (37)$$

In view of Eq. (36), $H(\zeta)$ is constant in the limit $\zeta \to \infty$. The integral in Eq. (37) therefore diverges as $\zeta^{2/3}$ for large arguments; accordingly, for $\omega \to \infty$

$$p(\omega) \sim \omega^{-2} c^{-5} U^9 D(\omega L/U)^{2/3} \sim \omega^{-4/3} U^{25/3} D^{5/3} \qquad (38)$$

Note that the divergence influences the scaling exponent, which therefore cannot be obtained by dimensional analysis alone. For small arguments, $H(\zeta) \sim \zeta^{16/3}$, therefore at low frequencies,

$$p(\omega) \sim D\omega^{-2} U^9 (\omega L/U)^6 = \omega^4 U^3 D L^6 \qquad (39)$$

Summarizing the results of Eqs. (38) and (39),

$$p(\omega) \sim c^{-5} \begin{cases} U^3 D^7 \omega^4 & \omega \sim 0 \\ U^{25/3} D^{5/3} \omega^{-4/3} & \omega \sim \infty \end{cases} \qquad (40)$$

Comparing Eqs. (40) and (29), it is evident that the ω^2 contributed by the fully developed region is dominant at low frequencies. Combining the contributions from the exit and fully developed regions,

$$p(\omega) \sim c^{-5} \begin{cases} U^5 D^5 \omega^2 & \omega \sim 0 \\ U^{25/3} D^{5/3} \omega^{-4/3} & \omega \sim \infty \end{cases} \qquad (41)$$

The conclusion that the fully developed region dominates the low frequency regime will be seen to agree with the classical theory of jet noise. But unlike the classical theory, the

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high frequency region in Eq. (41) contains inertial range contributions from both the fully developed and the exit regions of the jet.

V. Comparison with the classical theory of jet noise

An elementary scaling theory of jet noise[12] results from the additional hypothesis that each cross-section of the jet emits sound at one dominant frequency. Dimensional analysis based on the scaling laws Eqs. (19) and (30)-(31) at once implies that the contribution of the exit region to the acoustic frequency spectrum scales as ω^{-2} and that the contribution of the fully developed region scales as ω^2; analysis of Lighthill's integrals as in Sect. IV is not required. These are the well-known +2 and -2 scaling laws.

A refinement of this analysis is due to Lilley,[11] who argues that although the contribution to the frequency spectrum from each cross-section extends over a finite range of frequencies, the observed spectrum is the envelope of these contributions, and again scales as ω^{-2}.

These results will be rederived by suitably simplifying the time correlation functions. This calculation based on Lighthill's integrals will also provide a slight refinement since it provides formulas valid for all frequencies; it will be possible to show explicitly that the contribution of the fully developed region is vanishingly small for large ω and that the contribution of the exit region scales as ω^4 for small ω.

The fundamental idea of the classical theory can be incorporated in the present analysis by suppressing the wavenumber dependence of the time correlation functions; thus, Eq. (22) for the fully developed region is replaced by

$$\frac{1}{A(y)} \int dx dz \ R^{(0)}(\omega/Vp) * R^{(0)}(\omega/Vp) = \frac{y^2}{\mathcal{M}^{1/2}} F(\frac{\omega y^2}{\mathcal{M}^{1/2}}) \tag{41}$$

and Eq. (34) for the exit region is replaced by

$$\frac{1}{A(y)} \int dx dz \ R^{(0)}(\omega/Vp) * R^{(0)}(\omega/Vp) = \frac{y}{U} F(\frac{\omega y}{U}) \tag{42}$$

Thus, we effectively assume the separation of variables form Eq. (1) for the space time correlation function.

Repeating the calculation leading to Eq. (23),

$$p(\omega, y) \sim \omega^4 c^{-5} \mathcal{M}^{3/2} y^{-4/3} \int_{1/y}^{\infty} p^{-16/3} F(\omega y^2 \mathcal{M}^{-1/2})$$

$$= \omega^4 c^{-5} \mathcal{M}^{3/2} y^3 F(\omega y^2 \mathcal{M}^{-1/2}) \tag{43}$$

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Integrating Eq. (43) over y leads to

$$p(\omega) \sim \omega^4 c^{-5} \mathcal{M}^{3/2} \int_L^\infty dy \ y^3 F(\omega y^2 \mathcal{M}^{-1/2})$$

$$= \mathcal{M}^{5/2} \omega^2 \int_{\omega L^2 \mathcal{M}^{-1/2}}^\infty d\xi \ \xi F(\xi) \tag{44}$$

The integral in Eq. (44) converges when $\omega = 0$; accordingly, for small ω,

$$p(\omega) \sim \omega^2 \tag{45}$$

in agreement with the "+2" law. For large ω, assuming rapid decay of the time correlation function F for large arguments, instead

$$p(\omega) \sim 0 \tag{46}$$

.

In the near exit region, Eq. (42) leads to

$$p(\omega, y) \sim \omega^4 c^{-5} U^3 y^5 DF(\omega y/U) \tag{47}$$

Integrating Eq. (47) over y,

$$p(\omega) = \omega^4 c^{-5} U^3 D \int_0^L dy \ y^5 F(\omega y/U)$$

$$= \omega^4 c^{-5} U^3 D(U/\omega)^6 \int_0^{\omega L/U} d\xi \ \xi^5 F(\xi) \tag{48}$$

Again assuming sufficiently rapid decay of the time correlation function F at infinity, the integral converges as $\omega \to \infty$ leading to

$$p(\omega) \sim U^9 D\omega^{-2} \tag{49}$$

for large ω, in agreement with the -2 law. For small ω, the integral is of the order $(\omega L/U)^6$; consequently,

$$p(\omega) \sim \omega^4 \tag{50}$$

for small ω. Combining Eqs. (45), (46), (49), and (50), we recover the classical scalings

$$p(\omega) \sim \begin{cases} \omega^2 & \omega \sim 0 \\ \omega^{-2} & \omega \sim \infty \end{cases} \tag{51}$$

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VI. Frequency spectrum of sound radiated by shear turbulence

This section investigates the effects of shear on turbulent time correlations and the consequences of these effects for the acoustic power spectrum. The emphasis on time correlation effects should be stressed; previous investigations of sound radiation from shear turbulence have analyzed instead the effects of shear on directivity patterns, whether these effects are due to local anisotropy,[21] or due to mean flow effects.[7]

The effects of shear on time correlations will be evaluated by applying Leslie's perturbative theory of shear turbulence.[14] In this theory, the shear is treated as a formally small perturbation of a background state of isotropic turbulence; one can compare also the two-scale theory of Yoshizawa.[22] The result of the analysis is a perturbation series of the form

$$Q_{ij}(\mathbf{k}, t, s) = Q_{ij}^{(0)}(\mathbf{k}, t, s) + Q_{ij}^{(1)}(\mathbf{k}, t, s) + Q_{ij}^{(2)}(\mathbf{k}, t, s) \cdots \qquad (52)$$

where $Q^{(p)}$ is of order p in the mean velocity gradient. The single time correlation was computed to first order by Leslie;[14] for the corresponding calculation to second order, see Ref. 23. Corresponding to Eq. (52) is the series for the acoustic spectrum

$$p(\omega) = p^{(0)}(\omega) + p^{(1)}(\omega) + p^{(2)}(\omega) + \cdots$$

where $p^{(0)}(\omega)$ is the spectrum of isotropic turbulence.

Substituting the perturbation series Eq. (52) in the expression for acoustic radiation,

$$p^{(1)}(\omega) = \frac{\omega^4}{16\pi^2 c^5} \Delta_{ijkl} \int d\mathbf{p} \int_{-\infty}^{\infty} d\tau \, e^{i\omega\tau} \times$$
$$\left\{ Q_{ik}^{(1)}(\mathbf{p}, \tau) Q_{jl}^{(0)}(-\mathbf{p}, \tau) + Q_{il}^{(1)}(\mathbf{p}, \tau) Q_{jk}^{(0)}(-\mathbf{p}, \tau) \right.$$
$$\left. + Q_{ik}^{(0)}(\mathbf{p}, \tau) Q_{jl}^{(1)}(-\mathbf{p}, \tau) + Q_{il}^{(0)}(\mathbf{p}, \tau) Q_{jk}^{(1)}(-\mathbf{p}, \tau) \right\} \qquad (53)$$

The index symmetry of Ribner's factor implies that Eq. (53) can be rewritten as

$$p^{(1)}(\omega) = \frac{\omega^4}{16\pi^2 c^5} \Delta_{ijkl} \int d\mathbf{p} \int_{-\infty}^{\infty} d\tau \, e^{i\omega\tau} \frac{1}{6} \sum_{(ijkl)} Q_{ik}^{(1)}(\mathbf{p}, \tau) Q_{jl}^{(0)}(-\mathbf{p}, \tau)$$

where $\sum_{(ijkl)}$ denotes summation over all index permutations. The factor of 1/6 arises from the four terms in Eq. (53) and the 24 permutations in $\sum_{(ijkl)}$. By substituting the

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expression for $Q_{ij}^{(1)}(\mathbf{p}, \tau)$ from the Appendix, a simple calculation shows that

$$
\sum_{(ijkl)} \oint dS(\mathbf{p}) \, Q_{ik}^{(1)}(\mathbf{p}, \tau) Q_{jl}^{(0)}(\mathbf{p}, \tau)
$$

$$
= -\frac{32}{15}[R^+(p, \tau) + R^-(p, -\tau)]R^{(0)}(\tau)Q(p)^2 \sum_{(ijkl)} S_{ij}\delta_{kl} \tag{54}
$$

The time correlation functions R^+ and R^- in Eq. (54) are defined in the Appendix.

Assuming forms for the time correlation functions $R^{(0)}, R^+, R^-$ consistent with the sweeping hypothesis leads to

$$
p^{(1)}(\omega) \sim \mathcal{I}^1(S_{ij}) c^{-5} \varepsilon^{4/3} V^{13/3} \omega^{-7/3} \tag{55}
$$

where

$$
\mathcal{I}^1(S_{ij}) = \Delta_{ijkl} \sum_{(ijkl)} S_{ij}\delta_{kl}
$$

is a linear invariant of the rate of strain tensor, invariance being understood with respect to the statistical symmetries of the flow.

The perturbation series can be continued in principle to higher order terms in the mean velocity gradient. Because the results are very lengthy, let us attempt to generalize Eq. (55) heuristically. It is noted in the Appendix that whereas the single time correlation function $Q_{ij}^{(1)}(\mathbf{k}, t, t)$ can only depend on the symmetric part S_{ij} of the mean velocity gradient, the two time quantity $Q_{ij}^{(1)}(k, t, s)$ depends in general both on S_{ij} and the antisymmetric part

$$
W_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)
$$

Although the index symmetry of Ribner's factor Δ_{ijkl} prevents dependence of $p^{(1)}(\omega)$ on W_{ij}, $p^{(2)}(\omega)$ certainly can depend on both isotropic invariants $S_{ij}S_{ij}$ and $W_{ij}W_{ij}$. Dependence of the Reynolds stress itself on $W_{ij}W_{ij}$ can be ruled out on general invariance grounds.[16] Thus, it is possible that acoustic radiation by shear turbulence cannot be parametrized in terms of the Reynolds stress. From the viewpoint of this paper, such dependence would be problematic in any case since the Reynolds stresses are not associated with any temporal decorrelation mechanism. We conclude that

$$
p^{(2)}(\omega) \sim \mathcal{I}^2(S_{ij}, W_{ij})\omega^{-10/3} \tag{56}
$$

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The approximate calculation of jet noise of Sect. IV can be repeated using the second order perturbation series Eq. (52) to describe the sound source. Like the result for isotropic turbulence statistics, the results to first and second order will coincide with Eqs. (55) and (56). A turbulent jet has the symmetry type of axisymmetric shear flow in which S_{ij} is of the form

$$\begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & S_{23} \\ 0 & S_{23} & S_{11} \end{bmatrix} \tag{57}$$

so that $2S_{11} + S_{22} = 0$. The largest velocity gradient in a turbulent jet is the shear $\partial U_2 / \partial x_1$ in the near exit region. If this term dominates noise production, then in view of the structure of Eq. (57), $\mathcal{I}^1(S_{ij}) \sim S_{22} \sim 0$ but $\mathcal{I}^2(S_{ij}) \sim S_{12}^2$ and the shear contribution to the acoustic power spectrum will be given by Eq. (56) with $\omega^{-10/3}$ scaling.

These results for the spectra of sound radiated by isotropic turbulence and by shear turbulence combine to determine the complete pattern of acoustic radiation through the known directivities[7] of sound radiated by isotropic and axisymmetric turbulence. At 90 degrees to the jet axis, the shear contribution is entirely suppressed, and only the isotropic contribution, which scales theoretically as $\omega^{-4/3}$, should be observed. The effect of shear is greatest for nearly on-axis measurements; acoustic spectra measured in these directions should scale as $\omega^{-10/3}$. This very simple description of jet noise spectra is qualitatively consistent with the comprehensive survey of jet noise data of Tam et al,[17] who find that nearly on-axis spectra scale as $\omega^{-\alpha}$ with $\alpha \sim 2.8$. Unlike the 90 degree spectra, the nearly on-axis spectra exhibit unambiguous power law scaling. Of course, this comparison is only qualitative; more precise comparisons will depend on numerical simulations now in progress in which the present theory is used to characterize the sound sources.

We stress that in this account of jet noise, the "two components of turbulent mixing noise" described by Tam et al are consequences of the isotropic and strain dependent parts of the space-time correlation function of shear turbulence. To this extent, our theory remains a quadrupole theory, although coherent structures, which are prominent in the theory developed by Tam et al, enter crucially as the mechanism of temporal decorrelation. We propose that the failures of quadrupole theories described by Tam et al might be attributed not to inadequacies of these theories as such, but to the incorrect treatment of turbulent time correlations with which they have been applied.

VII. Conclusions

Proper parametrization of turbulent time correlations is crucial to calculation of power spectra of acoustic radiation from turbulent flows. The present suggestion, based on the sweeping hypothesis for Eulerian time correlations has been applied to the simplified classical theory of jet noise.

This theory could be incorporated in computer simulations in two ways. First, it permits stochastic synthesis of noise sources for linearized Euler calculations.[24] A second possibility is to incorporate these models in large eddy simulations as noise sources corresponding to the subgrid scales.

Finally, we note that our results favor the computational characterization of noise sources by methods capable of capturing the properties of coherent flow structures like large eddy simulation because the sweeping velocity depends on coherent flow structures.

Appendix. Leslie's theory of shear turbulence

We summarize the application of Leslie's perturbation theory of shear turbulence to the two time correlation function in shear turbulence.

We will consider a time stationary shear turbulence. This requires that energy be removed from large scales by some external agency; otherwise, as Kraichnan predicted[14] and experiments[25] later confirmed, the shear will continually create new scales; in the log layer, the external agency is turbulent diffusion.

Write the equation for velocity fluctuations in shear turbulence as

$$\frac{\partial}{\partial t}\,u_i(\mathbf{k},t) = -\frac{i}{2}P_{imn}(\mathbf{k})\int_{\mathbf{k}=\mathbf{p}+\mathbf{q}} d\mathbf{p}d\mathbf{q}\,u_m(\mathbf{p},t)u_n(\mathbf{q},t) + \mathcal{S}_{im}(\mathbf{k},t)u_m(\mathbf{k},t) \qquad (58)$$

where

$$P_{imn} = k_m P_{in}(\mathbf{k}) + k_n P_{in}(\mathbf{k})$$

$$P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j k^{-2}$$

and the mean shear operator is

$$\mathcal{S}_{im}(t) = -\frac{\partial U_i}{\partial x_m}(t) + 2k^{-2}k_i k_p \frac{\partial U_p}{\partial x_m}(t) + \delta_{im}k_s \frac{\partial U_s}{\partial x_r}(t)\frac{\partial}{\partial k_r}$$

Leslie writes the solution of Eq. (58) as a perturbation series

$$u_i(\mathbf{k},t) = u_i^{(0)}(\mathbf{k},t) + u^{(1)}(\mathbf{k},t) + \cdots$$

where $u^{(n)}$ is of order n in the strain rate and the zero order field is isotropic turbulence. Leslie's analysis assumes that the response function[1,14,15] $G^{(0)}$ of isotropic turbulence is known. If the shear is treated as a perturbation, then properties of the response function imply that the first order velocity field is

$$u_i^{(1)}(\mathbf{k}, t) = \int_0^t ds \ G^{(0)}(k, t, s) \ \mathcal{S}_{im} \ (\mathbf{k}, s) \ u_m^{(0)} \ (\mathbf{k}, s) \tag{59}$$

The two time correlation function in weakly sheared turbulence can be evaluated from Eq. (59). To first order,

$$Q_{ij}^{(1)}(\mathbf{k}, t, s)\delta(\mathbf{k} + \mathbf{k}') = < u_i^{(1)}(\mathbf{k}, t)u_j^{(0)}(\mathbf{k}', s) \ + \ u_i^{(0)}(\mathbf{k}, t)u_j^{(1)}(\mathbf{k}', s) >$$

Then

$$Q_{ij}^{(1)}(\mathbf{k}, t, s) = P_{ij}^{(1)}(\mathbf{k}, t, s) + \Pi_{ij}^{(1)}(\mathbf{k}, t, s) + T_{ij}^{(1)}(\mathbf{k}, t, s) \tag{60}$$

where

$$P_{ij}^{(1)}(\mathbf{k}, t, s) = -\int_0^t dr \ G(k, t, r)\frac{\partial U_i}{\partial x_m}Q_{mj}(\mathbf{k}, r, s) - \int_0^s dr \ G(k, s, r)\frac{\partial U_j}{\partial x_m}Q_{mi}(\mathbf{k}, r, t)$$

$$\Pi_{ij}^{(1)}(\mathbf{k}, t, s) = 2\int_0^t dr \ G(k, t, r)k_i k_p k^{-2}\frac{\partial U_p}{\partial x_m}Q_{mj}(\mathbf{k}, r, s)$$
$$+ 2\int_0^s dr \ G(k, s, r)k_j k_p k^{-2}\frac{\partial U_p}{\partial x_m}Q_{mi}(\mathbf{k}, r, t)$$

$$T_{ij}^{(1)}(\mathbf{k}, t, s) = \int_0^t dr \ G(k, t, r) \ < u_j^{(0)}(-\mathbf{k}, s)k_p \frac{\partial U_p}{\partial x_r}\frac{\partial}{\partial k_r}u_i^{(0)}(\mathbf{k}, r) >$$
$$+ \int_0^s dr \ G(k, s, r) \ < u_i^{(0)}(-\mathbf{k}, t)k_p \frac{\partial U_p}{\partial x_r}\frac{\partial}{\partial k_r}u_j^{(0)}(\mathbf{k}, r) >$$

The time stationary forms for $P_{ij}^{(1)}$ and $\Pi_{ij}^{(1)}$ are

$$P_{ij}^{(1)}(\mathbf{k}, \tau) = -R^-(k, \tau)\frac{\partial U_i}{\partial x_m}Q(k)P_{mj}(\mathbf{k}) - R^+(k, \tau)\frac{\partial U_j}{\partial x_m}Q(k)P_{mi}(\mathbf{k}) \tag{61}$$

$$\Pi_{ij}^{(1)}(\mathbf{k}, \tau) = 2R^-(k, \tau)\frac{\partial U_p}{\partial x_m}k_i k_p k^{-2}Q(k)P_{mj}(\mathbf{k})$$
$$+ 2R^+(k, \tau)\frac{\partial U_p}{\partial x_m}k_j k_p k^{-2}Q(k)P_{mi}(\mathbf{k}) \tag{62}$$

where

$$R^-(k, \tau) = \int_0^\infty dr \ G(k, r)R^{(0)}(k, r - \tau)$$
$$R^+(k, \tau) = \int_0^\infty dr \ G(k, r)R^{(0)}(k, r + \tau)$$

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where $R^{(0)}$ is the time correlation function of isotropic turbulence.

To evaluate correlations of the form $T^{(1)}$, define

$$A_{ijr}(\mathbf{k},t,s) = <u_i(\mathbf{k},t)\frac{\partial}{\partial k_r}u_j(-\mathbf{k},s)>$$

so that

$$T_{ij}^{(1)}(\mathbf{k},t,s) = -\int_0^t dr\ G(k,t,r)k_p\frac{\partial U_p}{\partial x_r}A_{jir}(-\mathbf{k},s,r)$$

$$-\int_0^s dr\ G(k,s,r)k_p\frac{\partial U_p}{\partial x_r}A_{ijr}(-\mathbf{k},t,r) \tag{63}$$

Note the properties

$$k_i A_{ijr}(\mathbf{k}) = 0$$

$$k_j A_{ijr}(\mathbf{k}) = -Q_{ir}(\mathbf{k}) = -P_{ir}(\mathbf{k})Q(k)$$

$$A_{ijr}(-\mathbf{k}) = -A_{ijr}(\mathbf{k})$$

$$A_{ijr}(\mathbf{k}) + A_{jir}(\mathbf{k}) = \frac{\partial}{\partial k_r}Q_{ij}(\mathbf{k}) = \frac{\partial}{\partial k_r}P_{ij}(\mathbf{k})Q(k) \tag{64}$$

and the general form, required by isotropy,

$$A_{ijr}(\mathbf{k}) = k^{-1}A(\mathbf{k})\epsilon_{ijr} + k^{-2}B(\mathbf{k})k_i\delta_{jr} + k^{-2}C(\mathbf{k})k_j\delta_{ir}$$

$$+ k^{-2}D(\mathbf{k})k_r\delta_{ij} + k^{-4}E(\mathbf{k})k_ik_jk_r \tag{65}$$

It follows from substituting the general form Eq. (65) into the conditions Eq. (64) that the functions $A,\cdots E$ are uniquely determined so that

$$<u_i(\mathbf{k},t)\frac{\partial}{\partial k_r}u_j(-\mathbf{k},s)>=$$

$$-Q(k,t,s)k^{-2}k_jP_{ir}(\mathbf{k}) + \frac{1}{2}\frac{d}{dk}Q(k,t,s)k^{-1}k_rP_{ij}(\mathbf{k}) \tag{66}$$

Substituting Eq. (66) in Eq. (63) and using Eqs. (60)-(62), we obtain the time stationary result

$$Q_{ij}^{(1)}(\mathbf{k},\tau) = Q(k)\{-R^-(k,\tau)\frac{\partial U_i}{\partial x_m}P_{mj}(\mathbf{k}) - R^+(k,\tau)\frac{\partial U_j}{\partial x_m}P_{mi}(\mathbf{k})$$

$$+ 2R^-(k,\tau)\frac{\partial U_p}{\partial x_m}k_ik_pk^{-2}P_{mj}(\mathbf{k}) + 2R^+(k,\tau)\frac{\partial U_p}{\partial x_m}k_jk_pk^{-2}P_{mi}(\mathbf{k})$$

$$- R^-(k,\tau)k^{-2}k_pk_iP_{jr}(\mathbf{k})\frac{\partial U_p}{\partial x_r} - R^+(k,\tau)k^{-2}k_pk_jP_{ir}(\mathbf{k})\frac{\partial U_p}{\partial x_r}\}$$

$$+ \frac{1}{2}[\hat{R}^-(k,\tau) + \hat{R}^+(k,\tau)]k^{-1}k_rk_pP_{ij}(\mathbf{k})\frac{\partial U_p}{\partial x_r} \tag{67}$$

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where

$$\hat{R}^-(k,\tau) = \int_0^\infty dr \ G(k,r)\frac{d}{dk}Q(k,r-\tau)$$

$$\hat{R}^+(k,\tau) = \int_0^\infty dr \ G(k,r)\frac{d}{dk}Q(k,r+\tau)$$

The symmetry properties of the result in Eq. (67) are interesting. Note the result of spherical integration

$$\oint dS(\mathbf{k}) \ Q_{ij}^{(1)}(\mathbf{k},\tau) = -\frac{1}{10}Q(k)\{R^+(k,\tau) + R^-(k,\tau)\}S_{ij} - \frac{1}{6}Q(k)\{\hat{R}^+(k,\tau) + \hat{R}^-(k,\tau)\}S_{ij}$$
$$- \frac{1}{6}Q(k)\{\hat{R}^+(k,\tau) - \hat{R}^-(k,\tau)\}W_{ij} \qquad (68)$$

and the single time reduction

$$\oint dS(\mathbf{k}) \ Q_{ij}^{(1)}(\mathbf{k}) = -\frac{1}{5}Q(k)R^+(k,0)S_{ij} - \frac{1}{3}Q(k)\hat{R}^+(k,0)S_{ij}$$

Thus, whereas the single time integrated correlation function depends only on S_{ij}, a consequence of index symmetries alone, the corresponding two time quantity depends on both S_{ij} and W_{ij}. This dependence is possible because the odd parity of the function of time difference which multiplies W_{ij} in Eq. (68) insures the symmetry relation

$$Q_{ij}(\mathbf{k},\tau) = Q_{ji}(\mathbf{k},-\tau)$$

satisfied by the two-point two-time correlation function.

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