On Spatial Fairness of the 802.11 DCF Protocol and the Role of Directional Antenna

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Abstract— The IEEE 802.11 DCF protocol suffers from the spatial fairness problem where different nodes achieve different throughput due to their topological distributions. In this paper we extend Bianchi's Markov model of the DCF protocol to multihop WLANs with some simple topologies and study the behavior of the binary exponential backoff (BEB) algorithm under heavy load conditions. Our study is focused on the interaction between the BEB processes of nodes at different locations. We identified several courses of the fairness problem, and found the BEB processes of some nodes have significant advantage over some other nodes. The BEB algorithm provides an undesired positive feedback and worsens the fairness problem. We further study directional antenna as a means to improve network fairness, and find that fairness can be improved significantly with directional antenna without changing the BEB algorithm.

Keywords: IEEE 802.11, spatial fairness, directional antenna.

I. INTRODUCTION

Wireless local area networks based on the IEEE 802.11 standards [1] have become one of the most prevalent types of wireless networks. As an important part of the standards, the distributed coordination function (DCF) of the MAC protocol has been subject to intensive studies [2], [3], [4]. It has been found that the DCF protocol suffers from the fairness problem, i.e. some users get very poor service compared to other users. Unfairness in wireless networks can be caused by unequal channel qualities [5], spatial bias [6], end-to-end network traffic patterns [7] or total throughput maximization [8]. Only the MAC layer fairness problem introduced by network topology is studied in this paper. This type of fairness problem can be called spatial or topological fairness because it is related to the network topology. It occurs in multihop networks as well as multiple overlapping single hop WLANs (using the same channel). The latter situation is not uncommon given the limited number of channels (only 3 orthogonal channels in 2.4GHz band) and the density of today’s WLAN deployment. Because of the co-channel interference between different single hop WLANs, they form a bigger network with effective topology spanning multiple hops at the MAC layer, even the traffic is still single hop 1. Nodes at the overlapping coverage area of the WLANs suffer from lower throughput. If the access points (APs) are part of a bigger wireless infrastructure and the stations perform hand off when they roam from the coverage area of one AP to the other AP, they receive the worst service during the handoff.

The fairness problem has been studied previously [9], [10], [11], [12] [13], [14], [15], [16], [4]. With numerous publications on this topic, some key questions remain: how do the binary exponential backoff (BEB) processes at different nodes interact with each other in a multihop topology? What role does this interaction play in the fairness problem? The backoff process at a node is driven by how much success or failure the node sees when it tries to acquire the channel. When different nodes have different neighbors, they experience different contention environments. Their BEB processes interact by contending for the channel and adjusting their own backoff window sizes in response to the perceived contention environments. We extend Bianchi’s discrete time Markov model to some simple multihop networks to analyze the interaction between the BEB processes at

1In this paper we use the term ”multihop” to refer to the fact that the network topology expands beyond a single hop at the MAC layer. The traffic studied is still single hop. This is different than a network where the traffic is carried through multiple hops. When traffic is multihop, the rate of different hops are correlated, and the heavy load traffic model does not apply.
different nodes. We study some simple networks to keep the analysis tractable, but still be able to capture the essence of the problem and shed light on the dynamics of the BEB processes. We identify several courses of spatial unfairness. The key observations are as follows:

- Due to their different local topologies, different nodes have different opportunities to contend for the channel. Some nodes are blocked from accessing the channel more often than others;
- Some nodes may have to compete for the channel with more competitors than other nodes. This causes more collisions for these nodes;
- The above factors force the BEB processes of some nodes to back off more than the other nodes. This rewards nodes which already have higher throughput and punish those nodes which have lower throughput. It becomes an undesirable positive feedback and it aggravates the fairness problem;
- The fairness problem becomes worse when the data packet size increases.

Many works, including [10], [11], [12] [13], [15], [16], attempt to improve fairness by replacing the BEB algorithm with enhanced contention or scheduling algorithms. Given the popularity of the BEB algorithm, we try to answer the question: is it possible to improve fairness without changing the BEB algorithm?

Directional antenna is often used to improve the capacity of wireless networks. It focuses the transmission energy in a certain direction and reduces the unwanted interference, providing a form of space segmentation. When interference is reduced, we expect the fairness in the network to improve\(^2\). The behavior of the BEB algorithm is investigated when directional antennas are used and is compared with the omni-antenna case. We found that by reducing the interference, directional antenna provides nodes with more equal opportunities to contend for the channel. The BEB processes at different nodes are more balanced, and fairness can be improved significantly without changing the BEB algorithm.

The organization of the paper is as follows. In Section II, Markov models are developed for the networks in Figs.1.a and 1.b with omni antennas. We identify the sources leading to the fairness problem. In Section III, a Markov model is developed for the network in Fig.1.c where idealized directional antennas are used at the access points. By comparing the networks with and without directional antennas, we show that fairness can be greatly improved by directional antennas.

II. MARKOV MODELS OF SOME SIMPLE NETWORKS

A. The network topology

Consider the network in infrastructure mode in Fig.1.a. There are two access points (AP₁, AP₂) in the network, and the rest are stations. The network can be considered as two partially overlapping WLANs in infrastructure mode. In groups A,C,B (from left to right), there are \(N₁, N₂, N₃\) nodes. Nodes in each group are fully connected. The edges show the links at the MAC layer, and the colored circles show the association between the stations and the APs. (b) shows an ad hoc network spanning two hops. Nodes in each group are fully connected, and nodes in group C are connected to nodes in A and B. (c) shows the same network as (a), but the APs use directional antennas instead of omni-antennas.

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\(^2\)Another advantage of directional antenna is extended transmission range. This is ignored in this paper since it is unrelated to the fairness problem.
B. Markov chain model

We now describe a discrete time Markov model for the network in Fig.1.a. It is based on the discrete-time Markov chain model of the 802.11 DCF protocol developed in [2]. The fundamental of the DCF protocol is for a node to adjust its backoff window size based on the collision it experiences. Time is slotted, and the probability that a node contends for the channel in a slot in which it is not blocked is called its contention probability \( \tau \). The probability that a node sees a collision in a slot where it contends for the channel is called its conditional collision probability \( p_c \). According to [2], the window backoff process of an 802.11 node can be modeled as a Markov process, where time is discrete and a time step represents either an idle slot, a collision slot or a successful data transmission. Under the assumption that every packet collides with constant and independent probability \( \tau \), the conditional collision probability \( p_c \) is a function of \( \tau \). Consequently the contention process of an 802.11 node is an i.i.d. Bernoulli process with probability \( \tau \). In a multihop network, the conditional collision probability \( p_c \) experienced by a node depends on the network topology. The focus here is to investigate how the backoff process is affected by network topology. Compared with the single hop network studied in [2], this is more difficult for the following reasons. In a single hop network, all nodes have the same view of the medium and they have the same notion of discrete time. In other words, an idle slot appears to all nodes as an idle slot and all nodes decrease their backoff counters by 1 in this slot. The same is true for a collision slot or a successful data transmission slot. Under these assumptions, the network becomes an ALOHA network where the nodes contend for the channel with the BEB algorithm. How many collisions a node experiences determines how much it needs to back off, and this in turn affects its contention probability. With these assumptions, it is possible to use a notion of synchronous discrete time for all the nodes in the network. The duration of an idle or collision slot is taken as the discrete time step in the Markov model. The backoff timers at different nodes can still advance differently, depending on the medium occupancy of their respective channels. These assumptions, especially the one that an idle slot takes the same time as a collision slot, deviates the model from the real situation. But these assumptions do not change the nature of the BEB process at each node. This is because these assumptions only change the relative durations between different types of slots, meanwhile the backoff procedure at each node is independent of the actual physical time. This ensures the BEB procedure is accurately modeled.

The states of the Markov chain are the channel occupancy states or MAC states. The state \((S_A, S_C, S_B)\) represents the status of nodes in group \(A,C,B\) in a slot, where \(S_l = 1\) if a node in group \(l\) is transmitting a data packet, or \(S_l = 0\) otherwise (nodes in \(l\) could see an idle slot, a collision slot, or are blocked from contending for the channel), \(l \in (A,B,C)\). There are total 5 states: \((0, 0, 0), (1, 0, 0), (1, 0, 1), (0, 0, 1), (0, 1, 0)\). Because conflicts are always detected by the RTS/CTS mechanism, data transmissions do not collide. Therefore states like \(0,1,1\), \((1,1,0)\) or \((1,1,1)\) are banned. In state
(0,0,0), both spacial channels are not occupied and all nodes are allowed to contend. In state (1, 0, 0) or (0, 0, 1), one of the two channels (CH₁ or CH₂) is used for transmission by a node in area A or B, nodes in area C are blocked from contending, and nodes in the other non-overlapping area (B or A) are allowed to contend for the unused channel. In (0, 1, 0), a node in C is transmitting to either AP and both channels are occupied. State (1,0,1) is the favored state from the total throughput point of view because both channels are used for data transmission. By solving this Markov chain, we can find the stationary distribution of the MAC states, and the contention probabilities τ₁ and the collision probabilities $p_l^i$ for nodes in each group, $l \in \{A, B, C\}$. Because groups A and B are symmetric, let $τ_A := τ_B$ and $p_l^i := p_A^i = p_B^i$, and $τ_C := τ_C$, $p_l^i := p_C^i$. Define the degenerate Binomial distribution

$$p^B(i, n, τ) = \begin{cases} (1 - τ)^n, & i = 0; \\ nτ(1 - τ)^{(n-1)}, & i = 1; \\ 1 - (1 + (n - 1)τ)(1 - τ)^{(n-1)}, & i = 2; \\ 1, & i = -1, \end{cases}$$

representing the cases that out of n nodes all contending with probability τ, the probability that zero (i = 0), one (i = 1), two or more (i = 2) nodes actually send RTS packets in a slot where the channel is available, and i = -1 represents the case that these n nodes are not allowed to contend for the channel because they are blocked by other transmissions. An example of blockage is that nodes in C cannot contend in state (1,0,0). Let

$$p(i, j, k, τ_1, τ_2) = p^B(i, N_1, τ_1)p^B(j, N_2, τ_2)p^B(k, N_1, τ_1)$$

be the probability that in groups A,C,B, respectively there are i,j,k nodes requesting the channel. Approximate the Markov processes as homogeneous and adopt the short hand notation $P(s|s) := P(X_{n+1} = s|X_n = s)$ with n being the slot number, the state transition probabilities of this Markov chain are given by

$$P(1, 0, 0|0, 0, 0) = p(0, 0, 1|0, 0, 0)$$

$$= p(1, 0, 0, τ_1, τ_2) + p(1, 0, 2, τ_1, τ_2),$$

$$P(0, 0, 0|0, 1, 0) = p(0, 0, 0|0, 0, 1)$$

$$= pt[(1, -1, 0, τ_1, τ_2) + p(-1, -1, 2, τ_1, τ_2)],$$

$$P(1, 0, 1|0, 0, 0) = p(1, 0, 1|0, 0, 0)$$

$$= (1 - pt)p(-1, -1, 0, τ_1, τ_2),$$

$$P(1, 0, 0|0, 1, 0) = p(0, 0, 1|0, 0, 1)$$

$$= pt(1 - pt),$$

$$P(1, 0, 0|0, 0, 1) = p(0, 0, 1|0, 0, 0)$$

$$= pt(-1, -1, 1, τ_1, τ_2),$$

$$P(0, 1, 0|0, 0, 0) = p(0, 1, 0, τ_1, τ_2),$$

$$P(0, 0, 0|0, 1, 0) = pt,$$ 

$$P(1, 0, 1|0, 0, 0) = p(1, 0, 1, τ_1, τ_2),$$

$$P(0, 0, 0|1, 0, 0) = p^2,$$ 

$$P(0, 1, 0|1, 0, 0) = P(0, 1, 0|0, 0, 1)$$

$$= P(0, 1, 0|0, 0, 1) = P(1, 0, 0|0, 0, 1)$$

$$= P(1, 0, 1|0, 1, 0) = P(0, 1, 0|1, 0, 0) = 0,$$

and the diagonal terms $P(s|s)$ can be obtained from

$$P(s|s) = 1 - \sum_{st \neq s} P(s|s).$$

Solving this Markov chain gives the stationary distribution of the MAC states, $p_s(0, 0, 0), p_s(1, 0, 1), p_s(0, 1, 0),$ and $p_s(1, 0, 0) = p_s(0, 0, 1)$ because of symmetry. Keep in mind that these stationary distributions are still functions of the contention probabilities $(τ_1, τ_2)$. In [3],[16], the contention process is assumed to be independent of other processes and the contention probabilities are constant. But to adjust the backoff window size based on collision history is a key characteristics of the BEB algorithm, and this is precisely how the BEB processes at different nodes interact with each other. We try to capture this by studying the interaction between the collision process and the backoff process. The BEB process at an individual node can be modeled as a separate Markov chain with backoff window size and backoff stage as its states [2]. It depends on the collision probability experienced by a node when it tries to access the channel. Given the condition that a node contends for the channel in a slot where it is not blocked, the expected conditional collision probabilities of the nodes in groups A,B and group C are

$$p_1^A(τ_1, τ_2) = \frac{p_s(0, 0, 0)(1 - (1 - τ_1)^{N_1-1}(1 - τ_2)^{N_2}}{p_s(0, 0, 0) + p_s(0, 0, 1)} + \frac{p_s(0, 0, 1)(1 - (1 - τ_1)^{N_1-1})}{p_s(0, 0, 0) + p_s(0, 0, 1)},$$

$$p_2^A(τ_1, τ_2) = 1 - (1 - τ_1)^{N_1}(1 - τ_2)^{N_2}. \tag{1}$$

To incorporate the backoff stages of different nodes into the Markov model directly will increase its size significantly. To keep the model simple, we use the following observation: at its stationary distribution, the contention process of a node is an i.i.d. Bernoulli process, where the contention probabilities $(τ_1, τ_2)$ depend on the conditional collision probabilities $(p_1^A, p_2^A)$ through
the following equation [2]:

$$\tau_i = \Pi(p_i^c) = \frac{2(1 - 2p_i^c)}{(1 - 2p_i^c)(W + 1) + p_i^cW(1 - (2p_i^c)^m)}$$

$$= \frac{2}{1 + W + p_i^cW\sum_{k=0}^{m-1}(2p_i^c)^k}, \quad i = 1, 2. \quad (3)$$

The minimum contention window size $W$ and the maximal backoff stage $m$ are protocol parameters. For 802.11 DSSS, $W = 32$ and $m = 5$, and for 802.11a/g $W = 16$ and $m = 6$. The parameters from 802.11a/g are used in this paper. Eq. 3 allows us to incorporate the BEB process (modeled by the Markov chain of Fig.4 in [2]) at each node accurately into the model without including them into the Markovian states. It significantly reduces the size of the Markov chain without sacrificing the accuracy.

Equations 1, 2 and 3 form a set of fixed point equations. Using the same technique as in [17], one can show with Brouwer’s fixed point theorem that there exists a fixed point solution $(p_1^*, p_2^*, \tau_1^*, \tau_2^*)$. The system can be further simplified by using the (numerical) inverse function of Eq.3

$$p_i^c(t(\tau_i)) = \Pi^{-1}(\tau_i), \quad i = 1, 2, \quad (4)$$

where $p_i^c(t)$ is the conditional collision probability required for the BEB process of a node to contend in a slot with probability $\tau_i$. The system can be reduced to

$$\Delta p_i^c(\tau_1, \tau_2) = p_i^c(t(\tau_i)) - p_i^c(\tau_1, \tau_2) = 0, \quad i = 1, 2, \quad (5)$$

with only two variables $(\tau_1, \tau_2)$. The solution of Eq. 5, $(\tau_1^*, \tau_2^*)$, can be found numerically. Throughout our numerical experiments, we have always found $\Delta p_1^c(\tau_1, \tau_2)$, $\Delta p_2^c(\tau_1, \tau_2)$ to be smooth and monotonic, and Eq. 5 always exhibited a single unique solution. No multiple fixed points were observed. Fig.2 and 3 show typical shapes of the functions $\Delta p_i^c(\tau_1, \tau_2)$. The collision probabilities $(p_1^*, p_2^*)$ can be obtained by substituting $(\tau_1^*, \tau_2^*)$ into Eq. 1 and 2, or equivalently Eq. 4. From the stationary distribution of the MAC states $p_s^*(0, 0, 0)$, $p_s^*(1, 0, 1)$, $p_s^*(0, 1, 0)$, and $p_s^*(1, 0, 0) = p_s^*(0, 0, 1)$, the normalized throughput$^3$ for the network and for a node in areas A, B and C are given by

$$R_{network} = p_s^*(1, 0, 0) + p_s^*(0, 0, 1) + 2p_s^*(1, 0, 1)$$

$$+ p_s^*(0, 1, 0),$$

$$R_A = R_B = (p_s^*(1, 0, 0) + p_s^*(1, 0, 1))/N_1, \quad (7)$$

$$R_C = p_s^*(0, 1, 0)/N_2. \quad (8)$$

$^3$Normalized w.r.t. the physical layer transmission rate. A node transmitting data at all time has a throughput of 1.

C. Numerical results and analysis

Fig.4 shows the throughput of the entire network and nodes in different groups of a network with $N_1 = N_2 = 20$, as the average data packet size is increased from 2 to 40 slots. Note that the total throughput increases as the data packet size increases. This is because the relative overhead consumed by contention (idle and collision slots) decreases as the durations of successful data transmissions increase. However, the throughput for a node in area $C$ is significantly lower than the throughput for a node in areas $A$ or $B$, and it decreases with increased data packet size. The fairness problem becomes worse as packet sizes increases. This is because that nodes in area $A$ and $B$ have more opportunities to contend for the channel than nodes in $C$. Fig.5 show the stationary distribution of the MAC states. The state $(1,0,1)$ dominates all the other states. This maximizes the spatial reuse as both spatial channels are used for data transmission and is good for the total network throughput. Note that no nodes can contend in state $(1,0,1)$ or $(0,1,0)$. From a contention point of view, only states $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$ are interesting. Nodes in $A$ (or $B$) can contend in states $(0,0,0)$ and $(0,0,1)$ (or $(1,0,0)$), but nodes in $C$ can contend only in state $(0,0,0)$. 

![Fig. 2. An example of function \(\Delta p_1^c(\tau_1, \tau_2)\).](image1.png)

![Fig. 3. An example of function \(\Delta p_2^c(\tau_1, \tau_2)\).](image2.png)
The unequal contention opportunity is the first source of unfairness. As the packet size grows, increasingly it suffers higher expected collision probability and this leads to more backoff and lower contention probability. Before the data transmission terminates and the MAC state turns from (0,0,1) to (0,0,0), a node in A is likely to grab the channel and the state becomes (1,0,1). This is why \( p_s^*(1,0,1) \) increases, and \( p_s^*(0,1,0), p_s^*(0,0,0) \) decrease with packet size. As \( p_s^*(0,1,0) \) is directly related to the throughput for nodes in C (Eq. 8), \( R_C \) decreases with packet size.

Fig.7 shows the contention probabilities \( (\tau_1^*, \tau_2^*) \) and the collision probabilities \( (p_1^{c*}, p_2^{c*}) \). The difference between \( p_1^{c*} \) and \( p_2^{c*} \) reflect the different contention environments experienced by nodes in A, B and in C. When a node in A contends, it contends in state (0,0,1) most of the time and only competes for the channel with \( N_1 - 1 \) nodes in A. Only in state (0,0,1) it needs to compete for the channel with \( N_1 + N_2 - 1 \) nodes in A and C. Meanwhile a node in C always competes for channel with \( N_1 + N_2 - 1 \) nodes. Note that the total potential number of competitors are the same for a node \( A_i \) (competing for \( A_i \rightarrow AP_1 \)) and a node \( C_i \) (competing for \( C_i \rightarrow AP_1 \) or \( C_i \rightarrow AP_2 \)). Overall, a node in C suffers higher expected collision probability \( p_2^{c*} > p_1^{c*} \) and this leads to more backoff and lower contention probability \( \tau_2^* < \tau_1^* \). The BEB processes at nodes in A, B and C are very unbalanced in favor of A and B. Even in state (0,0,0) where all nodes can contend for the channel on an equal basis, a node in C is typically
backed off more heavily and is less likely to acquire the channel. This puts a node in $C$ at a further disadvantage compared to a node in $A$. The difference in $\tau_1^*$, $\tau_2^*$ is caused by the difference in $P_{A1}^c$, $P_{C1}^c$, so it is a side effect of the first source of unfairness.

The effect of network size has also been investigated. As the network size increases, all nodes experience more collisions and more backoff. Nodes in all groups show increasing collision probabilities $p_i^c$ and decreasing contention probabilities $\tau_i$. However, the stationary distribution of the MAC states do not change much after the network size reaches $N_1 = N_2 = 15$ (with the relative sizes of different groups fixed), so the total throughput for each group remains relatively unchanged. The fairness problem appears insensitive to the number of nodes in each group. These results are not shown here due to the space limits.

D. Verification of the Markov chain model

A simulation program was developed to verify the correctness of the analytical model for the network in Fig.1.a. The contention process of each node follows the BEB algorithm. The simulation results compare favorably with the analytical results (Fig.8). Simulators have also been built for the other networks (Fig.1.b and 1.c) and similar agreements are found in all the cases.

E. Discussion of the homogeneous approximation

In the above model, we made the approximation that the contention probabilities $\tau_1^*$, $\tau_2^*$ do not change with time. This is true only if the collision probability experienced by a node is time-wise constant. For a node in $A$, it contends for the channel in states $(0,0,0)$ and $(0,0,1)$. Its RTS packet has different conditional collision probabilities in the two states. As the MAC state switches between $(0,0,0)$ and $(0,0,1)$, this node experiences two different contention environments and adjusts its backoff window size based on the different collision probabilities. From Fig.7 it can be seen that the highest contention probability is on the order of $3 \times 10^{-2}$. This means on average there are about 30 idle or collision slots in $CH_1$ between a node sends two consecutive RTS packets. The MAC state could switch between $(0,0,0)$ and $(0,0,1)$ multiple times during this interval, so a RTS packet sent by this node sees an averaged conditional collision probability given by Eq. 1. Therefore the approximation using the stationary distributions of the MAC states to compute $p_i^c$ and $\tau_i(p_i^c)$ can be justified.

F. The role of the number of competing nodes

Different nodes may need to contend for the channel with different numbers of contenders and consequently have different conditional collision probabilities. This further affects the size of the backoff window. This is the second source of unfairness. From the last example we know this is not the only cause of unfairness, because in Fig.1.a all nodes contend with the same number $(N_1 + N_2 - 1)$ of competitors. We now investigate this effect of different contending neighbors by studying the network in Fig.1.b. It is an ad hoc network with three groups of nodes. Nodes in each group are connected to the other nodes in the same group, and nodes in $C$ are further connected to nodes in $A$ and $B$. There are $N_1$, $N_1$ and $N_2$ nodes in groups $A, B, C$ respectively. Assume a node in a group only communicates with other nodes in the same group. In this network, a node in $C$ has to contend for the channel with all the nodes in the entire network (total $2N_1 + N_2 - 1$ competitors), while a node in $A$ or $B$ contends for the channel only with $N_1 + N_2 - 1$ competitors. The Markov model in Section II-B can be easily extended to model this network. The only change required is that Eq. 2 needs to be replaced by

$$p_i^c(\tau_1, \tau_2) = 1 - (1 - \tau_1)^{2N_1}(1 - \tau_2)^{N_2-1}. \quad (11)$$

Fig.4, 5 and 7 show the throughput, the stationary MAC states distribution and the contention/collision probabilities. The difference between the two networks is solely due to the increased number of contenders for nodes in $C$. The state $(0,1,0)$, which is the state that nodes in $C$ send data, diminishes further, and the fairness problem becomes worse.

Strictly speaking, these two courses of unfairness (unequal contention opportunities and unequal number of competitors) are not independent. The fundamental
reason for spatial unfairness is the interference between adjacent transmissions. The effect of these two courses contribute to each other through the BEB process. From Fig.7, it is clear that the increased competition for nodes in $C$ increases $p_{2}^{s}$ and decreases $\tau_{2}^{s}$. With nodes in $C$ contending less, nodes in $A$ and $B$ are more likely to acquire the channel, and $p_{2}^{s}(1,0,1)$ and $p_{1}^{s}(1,0,0), p_{2}^{s}(0,0,1)$ increase further. In the meantime state $(0,0,0)$ occurs less frequently, and the contention opportunity for nodes in $C$ reduces further. All these factors contribute to the diminished throughput for nodes in $C$. They produce a form of unwanted positive feedback through the BEB algorithm and makes the fairness problem worse than a system where all nodes contend with equal probability. It is not configured to favor the CTS from the AP treats it as a collision and backs off. We assume an idealized directional 802.11 MAC protocol is used at the APs. This means that nodes in the network do not suffer from the hidden nodes problem or the deafness problem [18]. When an AP is engaged in a data transmission in a sector, all the stations associated with this AP are aware of this transmission. The stations set their NAVs properly for the duration of this transmission, even if the transmission takes place in another sector (no deafness). Meanwhile, a station is not blocked unnecessarily. Denote nodes in $C$ associated with $A_{1}$ as subgroup $C_{1}$ and those associated with $A_{2}$ as $C_{2}$. A station in $C_{2}$ can contend for $A_{2}$ if $A_{1}$ is busy in $A_{1}$ but $A_{2}$ ($B_{1}^{s}, B_{2}^{s}$) is not busy. This is contrary to the network in Fig.1.a. In fact simultaneous use of sectors $A_{1}$ and $B_{2}^{s}$ enhances both total throughput and network fairness. With this idealized directional MAC protocol, we illustrate the full potential of directional antennas as a means to improve fairness.

As in the previous model, the states of Markov chain describe the MAC process of this network. A state $(S_{A}, S_{C_{1}}, S_{C_{2}}, S_{B})$, $S_{i} \in \{0,1\}$, $l \in \{A,C_{1},C_{2},B\}$, represents the situation that the channel is used by $S_{i}$ user in each group for data transmission in the slot. Because the AP has only one transmitter and can only transmit in one sector at a time, it is necessary that $S_{A} + S_{C_{1}} \leq 1$ and $S_{B} + S_{C_{2}} \leq 1$. Because the stations still use omni-directional antennas, it is also necessary that $S_{C_{1}} + S_{C_{2}} \leq 1$ (otherwise the two transmitting nodes in $C_{1}$ and $C_{2}$ will collide). The eight allowed states of the Markov chain are as follows: $(0,0,0,0),(0,0,0,1),(0,0,1,0), (0,1,0,0), (0,1,0,1), (1,0,0,0), (1,0,0,1), (1,0,1,0)$. We again use the approximation that the nodes in each group contend for the channel with constant probabilities, $\tau_{1} := \tau_{A} = \tau_{B}$ and $\tau_{2} := \tau_{C_{1}} = \tau_{C_{2}}$ because of symmetry. The conditional collision probabilities are $p_{i}^{c} := p_{i}^{c}_{A} = p_{i}^{c}_{B}$ and $\tilde{p}_{i}^{c} := \tilde{p}_{i}^{c}_{C_{1}} = \tilde{p}_{i}^{c}_{C_{2}}$. A node in group $A$ can contend for the channel in states $(0,0,0,0), (0,0,0,1)$ and $(0,0,1,0)$, and a node in $C_{1}$ can contend for the channel in states $(0,0,0,0)$ and $(0,0,0,1)$. The construction of the transition probability matrix of this Markov chain is similar to the previous one, but the number of terms becomes too large for enumeration. A MATLAB program was written to construct this transition matrix for given contention probabilities $(\tau_{1}, \tau_{2})$. For each MAC state, the allowed set of contention nodes $(i, j, k, l)$ in a slot, where $i,j,k,l \in \{-1,0,1,2\}$ are the number of nodes sending RTS in the four groups, are listed and the associated
probabilities are given by
\[ p_d(i, j, k, l, \tau_1, \tau_2) = p^B(i, N_1, \tau_1)p^B(j, N_2/2, \tau_2) \]
\[ \times p^B(k, N_2/2, \tau_2)p^B(l, N_1, \tau_1). \]

With the set of possible contention nodes in each MAC state, and the termination probability for the ongoing transmissions, a set of possible MAC states and related transition probabilities can be found. This way the transition probability matrix of the MAC states can be constructed numerically with a program. The conditional collision probabilities for a node in A, B and C1,2 are:

\[
P_1(\tau_1, \tau_2) = \frac{p_s(0, 0, 0, 0)}{P_{Ad}} (1 - (1 - \tau_1)^{N_1-1} \times (1 - 0.25N_2\tau_2(1 - \tau_2)^{N_2-1}))
\]
\[
+ \frac{p_s(0, 0, 0, 1)}{P_{Ad}} (1 - (1 - \tau_1)^{N_1-1} \times (1 - 0.25N_2\tau_2(1 - \tau_2)^{N_2/2-1}))
\]
\[
+ \frac{p_s(0, 0, 1, 0)}{P_{Ad}} (1 - (1 - \tau_1)^{N_1-1}),
\]

\[
P_2(\tau_1, \tau_2) = \frac{p_s(0, 0, 0, 0)}{P_{Cd}} (1 - (1 - \tau_2)^{N_2-1})
\]
\[
+ \frac{p_s(0, 0, 0, 1)}{P_{Cd}} (1 - (1 - \tau_2)^{N_2/2-1})
\]
\[
(1 - 0.5N_1(1 - \tau_1)^{N_1-1}),
\]

where

\[
P_{Ad} = p_s(0, 0, 0, 0) + p_s(0, 0, 0, 1)
\]
\[
+ p_s(0, 0, 1, 0),
\]

\[
P_{Cd} = p_s(0, 0, 0, 0) + p_s(0, 0, 0, 1),
\]

are the probabilities that a node in A or B, and in C is allowed to contend for the channel in any slot (contention opportunities). The dependency of the contention probabilities (\(\tau_1, \tau_2\)) on the conditional collision probabilities (\(p_1, p_2\)) is the same as before (Eq. 3). The set of equations (Eq. 3, 12, 13) can be solved numerically in the same way as in Section II-B. Like the case before, we have only found unique solutions of the fixed points throughout our experiments. Let \((p_1^*, p_2^*, (\tau_1^*, \tau_2^*) \) be the solution of the system. From the stationary distribution of the MAC states, the throughput are

\[
R_{network} = 2p_1^*(0, 1, 0, 0) + 2p_2^*(0, 1, 0, 0)
\]
\[
+ 4p_1^*(0, 1, 0, 0) + 2p_2^*(1, 0, 0, 1),
\]

\[
R_A = R_B = (p_1^*(1, 0, 0, 0) + p_2^*(1, 0, 0, 1)
\]
\[
+ p_1^*(1, 0, 1, 0))/N_1,
\]

The stationary distribution of the MAC states, network/node throughput, and contention/collision probabilities of the nodes are computed for variable data packet size and the results are shown in Fig.9, 10 and 11.

In Fig.10, the throughput of a node in C is within 10% of the throughput of a node in A or B. The relative ratio between them does not change with the data packet size. This is more pleasing than the case in Fig.4. The network resource is distributed more evenly among the nodes in different regions while the total network throughput is sustained. Fig. 6 compares the contention opportunities for nodes in group A, B and C, \(P_{Ad}^a\) and \(P_{Cd}^a\), with their counterpart \(P_{Ad}^o\) and \(P_{Cd}^o\) in the network of Fig.1.a. It is clear that when directional antennas are used at the APs, nodes in A, B no longer have dominant contention opportunities. Fig. 11 shows the contention probabilities (\(\tau_1^*, \tau_2^*\)) and collision probabilities (\(p_1^*, p_2^*\)). Compared with the omni-case (Fig.7), the BEB processes are more balanced with directional antenna. Similar \(\tau_1^*, \tau_2^*\) and similar \(p_1^*, p_2^*\) guarantee that all nodes get roughly equal share of the bandwidth. Comparing the
proportional fairness index in the two networks (Fig.12), we see that use of directional antenna makes the network significantly more fair.

IV. CONCLUSION AND FUTURE WORK

We have investigated the behavior of the BEB algorithms of the 802.11 DCF protocol in some simple networks with multihop topologies. Some sources of spatial unfairness have been identified at the MAC layer: unequal contention opportunities and unequal number of contending neighbors. These make the BEB process at different nodes unbalanced and force some nodes into more backoffs. The BEB algorithm provides a channel of unwanted positive feedback and makes the fairness problem worse. Through analyzing a network with idealized directional transmission, we found directional antenna can improve spatial fairness without changing the BEB algorithm. How to extend the current work to networks with general topology will be an issue of future research. It will also be interesting to see if fixed point solutions of the BEB processes in multihop networks are truly unique, and how fairness is improved by directional antenna changes in real systems.