

CS 417/517 Computational Methods and Software

Spring 2004

HW 1

Assigned: Thurs Jan 22, 2004; Due: Thurs Jan 29, 2004

You must show your work to receive credit for your answers. In problems where you are asked to give reasons, a numerical answer without a stated reason will receive no credit. This and future Homework problems will be posted in the course directory:

www.cs.odu.edu/~pothen/Courses/CS417. If any corrections are found to be necessary, they will be posted in this directory as well.

1. Do the following computations in Matlab (which uses the IEEE double precision floating point standard). Set `format long e` first, so you can see the results of the computations with 16 decimal digits. Write down the results you get when the following arithmetic expressions are calculated in Matlab. What results would you get if we could calculate each expression using real arithmetic? When Matlab computes answers that differ from these, explain why the results are different.
 - (a) $1 + 10^{-15}$
 - (b) $1 + 10^{-16}$
 - (c) $10^{200} * 10^{100}$
 - (d) $10^{200} * 10^{200}$
 - (e) $1/10^{300}$
 - (f) $1/10^{400}$
 - (g) $(4/3 - 1) * 3$ (calculate this expression exactly as it is written)
2. (This is not a Matlab problem.) Consider a floating point number system with base equal to 10, three decimal digits, and exponent in the range $-5 \leq e \leq 5$. Assume that the numbers are normalized, and that the rounding rule used is chopping.
 - (a) What is the overflow limit?
 - (b) What is the underflow limit? (Do not consider denormalized numbers.)
 - (c) What is the machine precision or unit roundoff?
 - (d) What is the largest difference between two successive numbers?
 - (e) What is the smallest difference between two successive numbers?
 - (f) What is the difference between zero and the smallest positive number?
 - (g) How many positive numbers are in this floating point system?
3. Suppose a and b are given numbers, and that we wish to calculate $c = \sqrt{a^2 + b^2}$.

- (a) First use the formula given to calculate the value of c in Matlab. Use the three pairs of values

$$a = 1, b = 1; \quad a = 10^{160}, b = 10^{160}; \quad a = 10^{-160}, b = 10^{-160}.$$

Explain the results you see.

- (b) Now consider the following equivalent computation in Matlab:

```
d = max(abs(a), abs(b)); a1 = a/d; b1 = b/d;
c1 = sqrt(a1^2 + b1^2); c = c1 * d
```

What values of c are computed for the values of a and b used in part (a)? Again, explain your results.

4. Study the two M-files `quadroot1.m` and `\quadroot2.m` for calculating the real roots of a quadratic equation $ax^2 + bx + c = 0$.

(These M-files are available in www.cs.odu.edu/~pothen/Courses/CS417.)

What roots do they compute when the following values of a , b , and c are used? If the answers computed by the two programs are different, explain the results.

(a) $a = 6, b = 5, c = -4$

(b) $a = 6 \times 10^{154}, b = 5 \times 10^{154}, c = -4 \times 10^{154}$

(c) $a = 1, b = 1, c = -1$;

(d) $a = 1 \times 10^{-170}, b = 1 \times 10^{-170}, c = -1 \times 10^{-170}$