

CS 417/517 Computational Methods and Software

Spring 2004

HW 3

Assigned: Thurs Feb 5, 2004; Due: Thurs Feb 12, 2004

Homework problems will be posted in the course directory:

www.cs.odu.edu/~pothen/Courses/CS417. If any corrections are found to be necessary, they will be posted in this directory as well. You must show your work to receive credit for your answers. In problems where you are asked to give reasons, an answer without a stated reason will receive no credit.

1. Alice buys three apples, twelve bananas, and one cantaloupe for \$2.36. Bob buys twelve apples and two cantaloupes for \$5.26. Carol buys two bananas and three cantaloupes for \$2.77. How much do single pieces of each fruit cost?
2. Matlab has a collection of ill-conditioned matrices that cause large errors in the computed solution when floating point arithmetic is used. These matrices include `magic`, `pascal`, `hilb`, etc. For this problem, let us investigate the $n \times n$ Hilbert matrix H , whose elements are given by

$$h_{ij} = 1/(i + j - 1), \quad \text{for } i = 1, 2, \dots, n; j = 1, 2, \dots, n.$$

- (a) In Matlab, type `type hilb.m` to see Matlab's efficient 'vectorized' code for computing the Hilbert matrix. There are five lines of code in this Matlab program. Explain what each line of code does.
Hint. Use Matlab's `help` command as you need. In particular, you might find `help punct` to give you useful information.
- (b) Create a Hilbert matrix of order n and create a right hand side vector \underline{b} such that the exact solution to $H\underline{x} = \underline{b}$ is the vector of all ones. Solve the system of equations $H\underline{x} = \underline{b}$ in Matlab. Compare the solution that you compute with the exact solution. Do this for three values of matrix sizes: $n = 10$, $n = 20$, and $n = 30$.
- (c) The difference between the computed solution and the exact solution (in absolute value) is known to be less than or equal to the product of two numbers: the condition number of the matrix and the machine precision. Use the Matlab command `cond` to calculate the condition numbers of the three Hilbert matrices; then compute the product of the condition number and machine precision in Matlab. Do the values of the product correctly predict the errors you see for the three values of n ?