

# CS 417/517 Computational Methods and Software

Spring 2004

Solutions to HW 6

Assigned: Thurs Mar 25, 2004; Due: Thurs Apr 1, 2004

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1. Matrix of normal equations =  $A' * A$

$$\begin{pmatrix} 5.0000 & 0 & 2.5000 \\ 0 & 2.5000 & 0 \\ 2.5000 & 0 & 2.1250 \end{pmatrix}.$$

Right hand vector of normal equations =  $A' * b$

$$\begin{pmatrix} 4.0000 \\ 1.0000 \\ 3.2500 \end{pmatrix}.$$

Cholesky factor is  $chol(A' * A)$

$$\begin{pmatrix} 2.2361 & 0 & 1.1180 \\ 0 & 1.5811 & 0 \\ 0 & 0 & 0.9354 \end{pmatrix}.$$

Least squares solution is

$$\begin{pmatrix} 0.0857 \\ 0.4000 \\ 1.4286 \end{pmatrix}.$$

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2. (a) The linear system is

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{pmatrix} x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

(b) The System  $QRx \approx b$  [Q R] = qr(A)

Q =

$$\begin{pmatrix} -0.5774 & 0.6172 & 0.5345 \\ -0.5774 & 0.1543 & -0.8018 \\ -0.5774 & -0.7715 & 0.2673 \end{pmatrix}.$$

and  $R =$

$$\begin{pmatrix} -1.7321 & -2.3094 \\ 0 & -2.1602 \\ 0 & 0 \end{pmatrix}.$$

So

$$\begin{pmatrix} -1.7321 & -2.3094 \\ 0 & -2.1602 \\ 0 & 0 \end{pmatrix} x = Q' * b = \begin{pmatrix} -3.4641 \\ -1.3887 \\ -0.2673 \end{pmatrix}.$$

(c)  $x =$

$$\begin{pmatrix} 1.1429 \\ 0.6429 \end{pmatrix}.$$

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3. c is the possible answer because it is orthongonal on every col of A.

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(a)

$$\begin{aligned} (Q_1 * Q_2) * (Q_1 * Q_2)^T &= (Q_1 * Q_2) * (Q_2^T * Q_1^T) \\ &= Q_1 * (Q_2 * Q_2^T) * Q_1^T = Q_1 * I * Q_1^T = Q_1 * Q_1^T = I \end{aligned}$$

(b) Since

$$\|Qv\|_2 = \|v\|_2$$

then

$$\|Q\|_2 = 1$$

Since

$$\|Q^{-1}\|_2 = \|Q\|_2 = 1$$

then

$$\text{cond}(Q) = 1$$