1. True or false? Give a reason for your answer to receive any credit.

(a) True. A linear least squares problem always has a solution. The solution is unique if \( A \) has a full rank; otherwise there are many solutions.

(b) True. Then the vector \( b \) lies in \( \text{span}(A) \).

(c) True. Householder matrices are orthogonal, and the product of orthogonal matrices is orthogonal.

(d) False. The normal equations approach squares the condition number of the matrix, and hence if the problem is badly conditioned, then the solutions will be inaccurate.

(e) True.

(f) True. The larger the angle (\( \theta \)), the larger the residual.

(g) True. Computing \( A^T A \) requires \( 2mn^2 \) flops, when \( A \) is an \( m \times n \) matrix. Orthogonal factorization methods requires \( 2mn^2 - \frac{2}{3}n^3 \) flops.

(h) True. Orthogonal matrices do not cause elements to grow.

2. \( a = \pm\sqrt{2} \).

3. (a) \( > [Q, R] = qr(A) \).

(b) \( > [Q, R, E] = qr(A) \).

4. (a) \( x = [1; 1] \).

(b) \( x = [7.009; -8.395] \).

Matrix \( A \) is badly conditioned: \( \text{cond}(A) = 1.0975 \times 10^3 \), since there are only three significant figures in the matrix elements of \( A \) and the elements of \( b \).