

# CS 417/517 Computational Methods and Software

Spring 2004

Assignment 7

Assigned: Thurs April 1, 2004; Due: Thurs April 8, 2004

1. True or false? Give a reason for your answer to receive any credit.
  - (a) True. A linear least squares problem always has a solution. The solution is unique if  $A$  has a full rank; otherwise there are many solutions.
  - (b) True. Then the vector  $b$  lies in  $\text{span}(A)$ .
  - (c) True. Householder matrices are orthogonal, and the product of orthogonal matrices is orthogonal.
  - (d) False. The normal equations approach squares the condition number of the matrix, and hence if the problem is badly conditioned, then the solutions will be inaccurate.
  - (e) True.
  - (f) True, The larger the angle ( $\theta$ ), the larger the residual.
  - (g) True, Computing  $A^T A$  requires  $2mn^2$  flops, when  $A$  is an  $m \times n$  matrix. Orthogonal factorization methods requires  $2mn^2 - \frac{2}{3}n^3$  flops.
  - (h) True. Orthogonal matrices do not cause elements to grow.
2.  $a = \pm\sqrt{2}$ .
3. (a)  $> [Q, R] = qr(A)$ .  
(b)  $> [Q, R, E] = qr(A)$ .
4. (a)  $x = [1; 1]$ .  
(b)  $x = [7.009; -8.395]$ .

Matrix  $A$  is badly conditioned:  $\text{cond}(A) = 1.0975 \times 10^3$ , since there are only three significant figures in the matrix elements of  $A$  and the elements of  $b$ .