

CS 417/517 Computational Methods and Software

Spring 2004

Assignment 7

Assigned: Thurs April 1, 2004; Due: Thurs April 8, 2004

1. True or false? Give a reason for your answer to receive any credit.
 - (a) A linear least squares problem always has a solution.
 - (b) If the right-hand side vector \underline{b} can be expressed as a linear combination of the columns of the matrix A , then the residual vector of the linear least squares problem $A\underline{x} = \underline{b}$ is the zero vector.
 - (c) The product of two or more Householder matrices is an orthogonal matrix.
 - (d) The normal equations approach to solving linear least squares problems computes accurate solutions irrespective of the condition number of the matrix.
 - (e) When the condition number of the matrix is large, orthogonal factorization methods must be used to compute the solution of a linear least squares problem.
 - (f) If the angle (θ) between the right-hand-side vector \underline{b} and the space spanned by the columns of A , $\text{span}(A)$, is large, then the residual vector of the linear least squares problem will also be large.
 - (g) When the number of observations (m) is much larger than the number of unknowns (n) in least squares problem, orthogonal factorization methods have roughly the same computational cost as methods based on Gaussian elimination.
 - (h) Row or partial pivoting is not used in computing the QR factorization, because orthogonal matrices do not cause elements to grow without bound during the computation.
2. Compute a Householder reflector H such that

$$H \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix}.$$

What is the value of a ?

3. Using Matlab, compute the QR factorization of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix}.$$

The first time use the `qr` command in Matlab that does not use column permutations. The second time use column permutations. Print the matrix Q and R in both cases.

4. Use Matlab to solve Computer Problem 3.4 from the book. What is the condition number of the matrix A in this problem? Explain the difference you see between the two solutions.