Abstract—The capacity of an ad hoc wireless network is determined by its topology and the resulting signal-to-interference ratio experienced by the various wireless links that interconnect different nodes. The Shannon capacity theorem is the fundamental result that relates the link bandwidth and the signal-to-interference ratio (SIR) experienced by a link. The link capacity in turn defines the upper bound on the data rate that a wireless link can support under the prevailing link conditions. The Shannon capacity of different links can be used to determine capacity regions that ensure stable link scheduling and link data rates. Since much of the research reported so far in this area has not dealt with exploiting concurrency that can be provisioned in the spatial domain using beamforming techniques, the links are forced to use transmission schedules that limit the network capacity. Assuming that each transmitter and/or receiver uses a multi-element transmit/receive antenna array, in this paper we describe a spatial processing technique that allows a receiver to concurrently form multiple high gain beams in the direction of different transmitters. Each beamformer can be designed in such a way that it provides high gain toward a single selected data source while nulling out the other sources. The resulting technique is shown to lead to higher traffic carrying capacity in a wireless network as a result of increased signal-to-interference ratio in the presence of multiple concurrent transmitters. While the proposed technique can be applied to any type of wireless network, the paper’s primary focus is on wireless sensor network topologies for supporting sensor data fusion by forming ad-hoc clusters of sensor nodes.

Keywords: Network capacity, Sensor networks, Capacity enhancement, Clustering, Concurrent beam-forming

I. INTRODUCTION

A wireless sensor network typically consists of many low cost individual sensor nodes that are dispersed in a given region. These nodes perform collaborative sensing by sending their individually sensed data to a special node that performs data fusion [3]. In ad-hoc sensor networks, one of the sensor nodes, called the cluster head (CH) may be selected or elected to take over the role of the fusion center [12] for a period of time. It is well known that if the sensed data contains useful information, then more sensed data sent to the CH will result in better quality of fusion. Since sensor nodes typically use wireless links for inter-node communication, we need to ensure that the underlying wireless network is able to support necessary network data traffic. The capacity of a wireless network defines its traffic carrying capabilities, and is determined by several factors [6], e.g., topology, coding, transmission power and signal-to-interference ratio. Several research papers [9], [16] have suggested techniques for finding bounds on the capacity as well as ways toward enhancing the capacity of a wireless network. In [16], the capacity is enhanced by designing a transmission schedule for the links so as to minimize mutual interference. However, for a cluster topology, the only meaningful schedule is to either sequentialize the transmission schedule or tolerate the adverse effects of mutual interference. In either case the result will be reduced network capacity. In this paper, we propose a technique that uses array antenna and associated beamforming techniques to introduce spatial concurrency to allow multiple nodes to transmit their data in parallel without causing any mutual interference. This is shown to result in significantly increased capacity of a cluster.

The paper is organized into four additional sections. Sec. II deals with identifying inherent capacity limitations in cluster topology. To address these limitations, Sec. III describes a beamforming structure that supports multiple and concurrent beamformers. Sec. IV describes the use of such a beamformer to enhance the capacity of a cluster. Conclusions and suggestions for future work are given in Sec. V.

II. CLUSTER TOPOLOGY CAPACITY LIMITATIONS

For an ad-hoc network of sensors, let \( s_1, s_2, \ldots, s_n \) denote the \( n \) nodes of such a network. Each node consists of a sensing mechanism, a wireless receiver, and a wireless transmitter. The sensor network formed by these nodes are required to communicate and collaborate with each other. Typically, the primary purpose of such collaboration is to facilitate sensor data fusion [3]. Sensor nodes communicate with each other using ad hoc networking protocols to elect one of the nodes as the cluster head based on some specified criterion [11]. Without loss of generality, we will assume that \( s_0 \) is designated as the cluster head.

The communication network is made up of radio links denoted as a wireless link \((i, j)\), such that \( s_i \) is the transmitting end and \( s_j \) is the receiving end. Let \( p_i \) denote the radio power level used by the transmitter of node \( s_i \) and let \( B \) denote the bandwidth of the radio link between any two nodes.
can reach a particular receiving end (i) simultaneous radio signals transmitted by other nodes that channel response matrix \( H \) in general radio signal. For a sensor network with \( n \) nodes, there are in general \( n \times n \) radio links that can be represented by a channel response matrix \( H = [h_{ij}]_{n \times n} \). The radio signal traveling over a particular link (i, j) is additively corrupted by: (i) simultaneous radio signals transmitted by other nodes that can reach a particular receiving end \( j, j = 1, \ldots, n; j \neq i \), and (ii) the broadband noise \( \nu_j \). The broadband noise \( \nu_j \) is modeled as zero mean white noise with Gaussian distribution \( N(0, \sigma_j^2) \), where \( \sigma_j^2 \) denotes the variance or the noise power spectral density. Collectively, the broadband noise is represented by the noise vector, \( \nu = [\nu_1, \ldots, \nu_n] \).

In a wireless network, generally there are arbitrary number of transmitting nodes that transmit simultaneously. Let the set of transmitting nodes be denoted by the subset \( \{ s_n : n \in n \} \). The network model as defined above can be used to compute the Shannon channel response function \( h_{ij} \) at the receiving end \( j \) of this link. The SIR is given by:

\[
\mu_{ij} = \frac{h_{ij}p_i}{\nu_j + \sum_{m \not\in n, m \not= i} h_{mj}p_m} \tag{1}
\]

Note that \( i \in n \) and \( j \not\in n \), because a wireless node can not transmit and receive simultaneously on a particular radio channel. The corresponding link capacity \( c_{ij} \) is then given by the well known Shannon capacity theorem [6],

\[
c_{ij} = B \log_2 (1 + \mu_{ij}) \tag{2}
\]

The goal then is to maximize the network capacity by maximizing individual link capacities \( c_{ij} \)’s. In the next section, we describe a technique that achieves this goal by introducing spatial concurrency.

## III. Multiple Concurrent Beamforming

Recent wireless networking technology based on the IEEE 802.11N [1] standards makes use of the multi-element antenna arrays in the either the transmitter, receiver or in both subsystems. Wireless nodes based on older technologies use a single-element transmit and receive antenna. Such antenna may be referred to as the Single Input Single Output (SISO) systems. Antennas with multiple elements in their transmit, receive or both paths are referred to as SIMO, MISO and MIMO, respectively [2]. An antenna with multiple elements results in increased antenna aperture that increases its directional sensitivity. In addition, the directional sensitivity of such multi-element antennas can be controlled as a function of direction. Controlled directional sensitivity translates into higher signal-to-interference ratio (SIR), which as per the Shannon capacity theorem results in greater wireless link capacity. Our immediate goal is to provide a solution for increasing the capacity of a cluster to allow its nodes to communicate the maximum amount of data to the cluster head. To keep focus on the issue of capacity enhancement, only the cluster head is assumed to have a multi-element beamforming array antenna in its receive path. The basic concept can be easily extended to the case in which a transmit side, (i.e., a cluster node) has a multi-element antenna too. The receive side array antenna is assumed to have \( K \)-elements and each element’s output is multiplied by a complex weight as shown in Fig. 1. These weights determine the spatial or directional response of an antenna and hence the channel gain. Assume that there are \( n \) wireless nodes \( s_0, \ldots, s_{n-1} \) in a cluster. The node \( s_0 \) is assumed to be the cluster head that aggregates the data transmitted by the cluster members \( s_1, \ldots, s_{n-1} \), as shown Fig. 1. Consider the link between a particular node \( s_i \) and the cluster head \( s_0 \). Without loss of generality, assume that these nodes are lying in a 2-D Cartesian space such that the \( i \)th node is located at coordinates \((a_i, b_i)\). We further assume that the distance between any two communicating nodes is several wavelengths, i.e., the transmitting node is assumed to be lying in the far field of the array antenna. The signal \( x_0 \) received by the receiving node’s antenna array can then be approximated as having a planar wave front that arrives at an angle \( \theta_i, i = 1, \ldots, n-1 \) to the array normal as shown in Fig. 1. Without loss of generality, in the remainder of the paper it is assumed that the network geometry and hence the direction-of-arrivals \( \theta_i \)’s are known for every node in this network. This assumption can be easily met either by letting each node find its own location and broadcast its position coordinates to other nodes or by using one of the well known Direction-of-Arrival (DoA) estimation algorithms like MUSIC [15] or ESPRIT [14]. The baseband signal received at the antenna array from the \( i \)th node can be modeled as a vector \( X \) of length \( n \) given by

\[
X^* = \left[ 1 \quad e^{j \phi_1} \quad e^{j 2 \phi_1} \quad \ldots \quad e^{j (K-1) \phi_1} \right] + V^* \tag{3}
\]

where \( ^* \) denotes conjugate transpose and \( \phi_i = \frac{2 \pi f d \sin(\theta_i)}{\lambda} \), \( \lambda \) is the corresponding wavelength. Commonly, the
inter-element spacing is \( d = \lambda/2 \) or \( d = \lambda/4 \). The random noise component is assumed to be Gaussian and uncorrelated between different elements of the receiving antenna array, i.e., \( E\{\nu_i^*\nu_j\} = \sigma^2\delta_{ij} \), where \( \sigma^2 \) is the noise power and \( \delta_{ij} \) is the Dirac delta function. Assuming \( d = \lambda/2 \) as the inter-element spacing, then \( \phi_i = \pi\sin(\theta_i) \). For now, let us consider that the cluster head is only interested in receiving data from a single node say \( i \), while all other nodes, if allowed to transmit, act as sources of interference\(^1\). This interference can be minimized by using an optimal beamforming algorithm that takes into account the directional and other attributes of the desired signal and the interferences. Numerous beamforming algorithms have been proposed in the literature (see for example [7], [8], [10]). The linearly constrained adaptive beamforming algorithm proposed in [7] is well suited for our needs. This algorithm treats the direction-of-arrival of the signal from the \( i \)th node as the desired signal or the look direction. The optimality criterion then is to minimize the total output power \( R_{yy} \) in the output signal \( y \), subject to the constraint that \( C_i^*W = 1 \). Here, \( C_i \) is called the look direction vector for source \( i \) and its conjugate transpose is given by

\[
C_i^* = \begin{bmatrix} 1 & e^{-j\phi_1} & \ldots & e^{-j(K-1)\phi_i} \end{bmatrix}
\]  

(4)

and \( W \) is weight vector to be computed by the beamformer based on the optimality criterion stated earlier and can be written as

\[
W^* = [w_0, w_1, \ldots, w_{K-1}]
\]  

(5)

The output of the beamformer is then given by,

\[
y(t) = W^*X(t) = \sum_{i=0}^{K-1} w_i^*x_i(t)
\]  

(6)

where \( t \) denotes the time index. The covariance matrix \( R_{yy} \) of the output signal \( y(t) \) can be written as,

\[
R_{yy} = E\{yy^*\} = W^*R_{xx}W + \sigma^2 I
\]  

(7)

The optimal beamforming problem can now be formally stated as a constrained optimization problem,

\[
\min_W R_{yy} \quad \text{subject to} \quad C_i^*W = 1
\]

(8)

The solution to the above constrained optimization problem is given by [7],

\[
W_{opt} = R_{xx}^{-1}C_i[C_i^*R_{xx}^{-1}C_i]^{-1}
\]  

(8)

From Eq. (8), it follows that \( W_{opt} \) requires (i) knowledge or the estimate of the autocorrelation matrix \( R_{xx} \), and (ii) matrix inversion operation, which is an \( O(N^3) \) operation. These overheads can be reduced by resorting to either iterative techniques [7] or using techniques like cascade beamforming that entail inverting only a 2x2 matrix [8]. Since the primary focus of this paper is on wireless network capacity enhancement rather than on the beamforming algorithms, we consider a simplified situation in which, (i) the additive white Gaussian noise (AWGN) is absent, and (ii) increasing the capacity of single link, say between node \( i \) and the cluster head. All other nodes \( j \neq i \) that are trying to send their data to the cluster head are treated as being interference by the beamformer and these nodes need to nulled out. In this case, the optimal beamforming problem reduces to a beam-pattern synthesis problem that requires computing the weight vector by solving the linear system of equations

\[
\begin{bmatrix}
1 & e^{j\pi\phi_1} & \ldots & e^{j(K-1)\phi_1} \\
1 & e^{j\pi\phi_2} & \ldots & e^{j(K-1)\phi_2} \\
\vdots & \vdots & \ddots & \vdots \\
1 & e^{j\pi\phi_{M-1}} & \ldots & e^{j(K-1)\phi_{M-1}}
\end{bmatrix}
\begin{bmatrix}
w_0 \\
w_1 \\
\vdots \\
w_{K-1}
\end{bmatrix}
= \begin{bmatrix} 1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

In general, \( M \neq K \) and such a matrix is not invertible. However, we can use the cascade structure described in [4] and [8] since it requires inversion of 2x2 matrices that has the additional benefit of exhibiting computational efficiency. It will be instructive to utilize the following observation for a 2-element antenna array. A 2-element antenna array has two degrees of freedom that can be exploited to steer the antenna in the direction of the desired source (i.e., node 1 for the above example) while simultaneously steering a null in the direction of one other interfering source. The two weights are computed to perform such an action. Mathematically, the weights for such a 2x2 antenna array can be computed by solving,

\[
\begin{bmatrix}
1 & e^{j\pi\phi_1} \\
1 & e^{j\pi\phi_2}
\end{bmatrix}
\begin{bmatrix}
w_0 \\
w_1
\end{bmatrix}
= \begin{bmatrix} 1 \\
0
\end{bmatrix}
\]

\[
w_0 = \frac{e^{j\pi\sin(\phi_2)} - e^{j\pi\sin(\phi_1)}}{e^{j\pi\sin(\phi_1)} - e^{j\pi\sin(\phi_2)}}
\]

\[
w_1 = \frac{1}{e^{j\pi\sin(\phi_1)} - e^{j\pi\sin(\phi_2)}}
\]

(9)

The first stage of a K-element array can then be viewed as having \((K-1)\) pairs of array elements with weight-pair \([w_{1,0} \ w_{1,1}]\), as shown in Fig. 2. Each pair of elements has identical weights and hence gives identical directional response. The output of the first stage can then be viewed as a \((K-1)\) vector. In the second stage of processing, the antenna array weights are computed to steer the null in the direction of another interfering source while maintaining the direction of node 1 as being the look direction. This cascading structure can be terminated at any level \( \leq (K-1) \). It is relevant to mention here that a null steered by a previous stage does not get undone by the next stage as explained in [4]. The overall effect of such beamforming is to minimize the contribution (i.e. interference) from all other cluster nodes except in the direction of node 1, which is constrained to a constant value of 1. As discussed previously in Section I, as per the Shannon capacity theorem, reduction in interference leads to higher SIR, which in turn translates into higher link capacity. The first stage operates on an input represented as an \( K = 4 \)-dimensional vector \([x_{1,0} \ x_{1,1} \ x_{1,2} \ x_{1,3}]^T\) by minimizing the contribution of signals emanating by node 2, while maintaining a constant gain toward the signal transmitted by node 1. The

\(^1\)Later on in this section, we describe the concurrent beamforming structure that allows multiple nodes to send useful data to the cluster head simultaneously.
output of the first stage can be represented by the \((n-1)\) vector 
\[Y_1 = [x_{2,0} \ x_{2,1} \ x_{2,2}]^T\] that acts as the input for second stage. Assuming that there are three nodes that want to send data to the cluster head, the output \(Y_1\) of the first stage receiving array is given by the vector,
\[
\begin{bmatrix}
    x_{2,0} \\
    x_{2,1} \\
    x_{2,2}
\end{bmatrix}
\begin{bmatrix}
    w_{1,0}^* \\
    w_{1,1}^* \\
    w_{1,2}^*
\end{bmatrix}
\begin{bmatrix}
    1 + 1 + 1 \\
    \sum_{i=1,2,3} e^{j\phi_i} \\
    \sum_{i=1,2,3} e^{j\phi_i}
\end{bmatrix}
\]
where, \(\theta_i = \pi \sin(\phi_i)\) and \(\phi_i, \ i = 1,2,3\) correspond to the directions of these three nodes. Expanding the right hand side gives,
\[
Y_1 = \begin{bmatrix}
    g_1 + g_2 + g_3 \\
    g_1 e^{j\phi_1} + g_2 e^{j\phi_2} + g_3 e^{j\phi_3} \\
    g_1 e^{j2\phi_1} + g_2 e^{j2\phi_2} + g_3 e^{j2\phi_3}
\end{bmatrix}
\]
where, \(g_1 = 1 = w_{1,0}^* + w_{1,1}^* e^{j\phi_1}\) is the unity directional gain experienced by node 1, \(g_2 = 0 = w_{1,0}^* + w_{1,1}^* e^{j\phi_2}\) is the zero directional gain experienced by node 2 and \(g_3 = w_{1,0}^* + w_{1,1}^* e^{j\phi_3}\) is some arbitrary complex gain experienced by node 3. The purpose of explicitly writing \(Y_1\) in this form is to highlight the fact \(Y_1\) has zeroed out node 2’s contribution and that direction induced phase difference between elements of \(Y_1\) has remained unaffected. Therefore, second stage processing treats \(Y_1\) as 3-element array that has to null out the contribution of node 3 arriving from direction \(\phi_3\). The second stage weights can be easily computed from Eq. (9) by substituting \(\phi_3\) in place of \(\phi_2\). Note that, for the present discussion, the cluster head treats node 1 as the primary data source while treating the other three transmitting nodes as interferences. The first stage of processing maximizes SIR with respect to source 2 by steering a null in the direction \(\phi_2\). The second stage weights are computed such that the array steers a null in the direction of the signal emanating from node 3, while maintaining a constant response toward node 1. Finally, the third stage steers a null toward node 3. Cascading these three stages as shown in Fig. 2 nulls out all the interfering transmissions, thus maximizing the SIR with respect to nodes 2, 3 and 4. This figure corresponds to the case in which the cluster head treats node 1 as the primary data source while other transmitting nodes (i.e., 2, 3 and 4) are treated as interferences. In general, if the cluster head has a \(K\)-element receiving array, the final output is capable of maximizing SIR with respect to \((K - 1)\) wireless nodes. The directional response of different beamforming stages as well as the composite beam pattern is as shown in Fig. 3. The cluster head treats node 1 as the primary data source while other nodes 2,3 and 4 are treated as interferences. The multi-stage beamformer achieves this by steering nulls in the direction of nodes 2,3 and 4 while maintaining a fixed response in the direction of node 1. This ensures that the beamformer is able to maximize the SIR at the cluster head with respect to node1, as desired. The cluster head can use

IV. CLUSTER CAPACITY ENHANCEMENT

Consider a sensor network of \(n\) nodes in which one of the sensor nodes functions as a cluster head (CH) while the remaining \(n-1\) sensor nodes send their sensed data to the CH. It is further assumed that each sensor node directly
of a cellular phone network, router/access-point of a WiFi network, etc. The capacity of a single-hop network or a cluster can be derived by computing the capacity region of the given network. The capacity region is shown to be the convex hull of the \( n - 1 \) basic data rate matrices corresponding to the \( n - 1 \) directional links. For a single hop network, the capacity region computed by the technique described in [16] either precludes multiple sensors transmitting their data to the CH to minimize mutual interference (i.e., following a sequential transmission schedule) or allowing multiple transmissions at the expense of increased mutual interference. In either case, the effect is to reduce the overall network capacity. Without loss of generality, assume that the 0th sensor node is designated as the CH. Then for a single hop network, the basic data rate matrix \( R_i = [r_{ij}] \) is an \( (n-1) \times (n-1) \) matrix with all its entries as 0 except for the two entries \( r_{0i} = r_{insc} \) and \( r_{ii} = -r_{insc} \), where \( r_{insc} \) is computed as:

\[
r_{i,nsc} = W \log_2 \frac{G_i P_i}{w} + \sum_{j=1, i \neq j}^{n-1} G_j P_j \quad (10)
\]

where \( i \) denotes the link between the \( i^{th} \) node and the cluster head, \( G_i \) is the corresponding channel gain, \( P_i \) is transmission power and \( w \) is white Gaussian noise modeled as \( N(0, \sigma^2) \). Since only a single link at a time is allowed to transmit its data to the CH, the capacity region \( CR_{nsc} \) (no spatial concurrency) is given by

\[
CR_{nsc} = \sum_{i=1}^{n-1} \alpha_i R_{i,nsc} \quad \sum_{i=1}^{n-1} \alpha_i = 1 \quad (11)
\]

where \( \alpha_i \) is fraction of the time for which the link \((0, i)\) is active and \( R_i \) matrices are formed as described above using the \( r_{i,nsc} \) values. Based on Eq. (11), the dotted line in Fig. 6 shows the capacity region \(^2\) for the sample network topology shown in Fig. 4. In this example, node 0 is chosen as the cluster head. Therefore, the fastest data rate \( r_i(max) \) that can be supported by various wireless links can be represented by an \( n - 1 \) vector that terminates on this convex hull. Let this vector be represented as \([\rho_1, \ldots, \rho_{n-1}]\) and the resulting network capacity is given by

\[
C_{nsc} \leq \sum_{i=1}^{n-1} \rho_i \quad (12)
\]

The exact value of \( C_{nsc} \) is determined by how the links are scheduled. The sequential schedule will permit each link to transmit data at its maximum rate, since there will be no mutual interference. However, the time taken by all sensor nodes to transport their data to the CH is the sum of individual transmission durations. In contrast, if multiple nodes are scheduled to transmit concurrently, the resulting mutual interference will reduce the individual link data rates, thus reducing the cluster capacity.

\(^2\)Fig. 6 shows a 2-D of the capacity region. The actual convex hull is an \( n - 1 \) dimensional polygon that cannot be easily visualized.
The capacity of this single-hop network can be increased beyond $C_{nsc}$ by introducing spatial concurrency with the help of concurrent beamforming described in Section III. The CH now employs an array antenna along with multiple beamformers that work concurrently. We first assume that the antenna array has $n-1$ elements, thus giving the beamformer $n-1$ degrees of freedom in computing the array beam-pattern. With $n-1$ degrees of freedom, the system can provide $n-1$ independent beamformers, such that the $i^{th}$ beamformer associated with the wireless link $l_{0,i}$, $i = 1, \ldots, n-1$, has a response that nulls out all other transmitting sources except the $i^{th}$ source. This implies that the link $l_{0,i}$ can now support a data rate given by

$$r_{i,sc} = W \log_2 \frac{G_i P_i}{W}$$  \hspace{1cm} (13)$$

even while all other nodes $j$, $j = 1, \ldots, n-1; j \neq i$ are also transmitting their data at a rate $r_{j,sc}$ to the CH simultaneously. As a result, all nodes can now transmit simultaneously and the CH is able to receive this data without causing mutual interference. The capacity region now is now an $n-1$ dimensional arc of radius $\sum_i r_{i,sc}$ as shown in Fig. 6 and the network capacity is given by

$$CR_{sc} = \sum_{i=1}^{n-1} r_{i,sc}$$  \hspace{1cm} (14)$$

It is easy to conclude from Fig. 6, that with the concurrent beamformer at the CH, the traffic carrying capacity of the cluster is significantly increased. The increased capacity comes at a price by way of having to provide an array antenna that has significantly increased processing and accompanying battery power requirements.

V. CONCLUSIONS & FURTHER RESEARCH

In this paper, we have described a technique to increase the capacity of a sensor network that is organized as a single-hop cluster to support sensor data fusion. The proposed technique utilizes an adaptive beamforming array to maintain desired gain in the direction of a single sensor node, while simultaneously nulling out interference from all other nodes. Providing $(n-1)$ such concurrent beamformers ensure that this advantage can be accrued with respect to all the $(n-1)$ sensor nodes, thereby giving a linear increase in the network traffic capacity for sensor data fusion applications.

The proposed technique may turn out to be impractical for a large cluster since it would require an array antenna having a large number of elements. Large values of $n$ (e.g., $n = 21$) would require CH to employ an antenna array that has 20 elements separated by a distance of $\lambda/2$. This can make the antenna physically and computationally impractical. A possible solution may be to use an antenna with smaller number of elements, e.g., $n' = 4$. This will imply that the CH will have only $n'$ concurrent beamformers. The network capacity analysis can now be done by modifying the combinatorial analysis outlined in [16]. Furthermore, a cluster does not need to be a 1-hop cluster always. A $k$-hop cluster as described in [5] will have the advantage of having wireless links that are shorter in terms of physical distance between the two ends. A shorter link will translate into better SIR due to reduced radio transmission power. The proposed technique can be modified to further increase the capacity of a $k$-hop cluster [5]. By treating the array size $K$ and the cluster depth $k$ as design parameters, it would also be relevant to analyze their effect on the traffic capacity and the quality-of-service (QoS) of a sensor network cluster. However, due to the size limitation these issues have been left out this paper. Instead, we plan to report more comprehensive results in another paper.

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