CS 381 Solutions to Test

February, 2005

1. Which ones of the following sentences are propositions and which ones are not ? [15]

(a) $x = y + 1$
No

(b) Is he happy ?
No

(c) If and only if he is healthy, he is happy.
Yes

(d) Either he is happy or he is not healthy.
Yes

(e) If he is happy, then he is healthy.
Yes

2. Convert the following propositions into "If_then_" form without changing their truth value. [15]

(a) Fuel savings follows from good insulation.
If insulation is good, then fuel is saved.

(b) There has been sufficient water only if the plant is healthy.
If there has been sufficient water, then the plant is healthy.

(c) A modification of the program is a necessary condition for the introduction of errors.
If errors are introduced, then the program has been modified.

3. Find the contrapositive of each of the following propositions: [15]

(a) Fuel savings follows from good insulation.
If fuel is not saved, then insulation is not good.

(b) There has been sufficient water only if the plant is healthy.
If the plant is not healthy, then there has not been sufficient water.

(c) A modification of the program is a necessary condition for the introduction of errors.
If the program has not been modified, then errors are not introduced.

4. Negate each of the propositions given below in English. Give a form **other than** simply putting 'not' or 'It is not the case that' in front or anything similar. [15]

(a) The processor is fast but the printer is slow.
The processor is not fast or the printer is not slow.

(b) If the processor is fast, then the printer is slow.
The processor is fast and the printer is not slow.

(c) Either the processor is fast and the printer is slow, or the file is damaged.
The processor is not fast or the printer is not slow, and the file is not damaged.

5. Fill in the blanks with the **shortest** string of characters so that the resultant proposition is valid. [20]

(a) $[P \to Q] \wedge \neg Q \Leftrightarrow [\boxed{\neg P} \vee Q] \wedge \neg Q$
$\Leftrightarrow [\boxed{\neg P} \wedge \neg Q\ ] \vee [Q \wedge \boxed{\neg Q}\ ]$
$\Leftrightarrow [\boxed{\neg P} \wedge \neg Q] \vee \boxed{\text{F}}$

(b) $[[P \to Q] \wedge [Q \to R]] \to [P \to R]$
$\Leftrightarrow [\neg \boxed{P \to Q} \vee \neg \boxed{Q \to R}\ ] \vee [P \to R]$
$\Leftrightarrow [[P \wedge \boxed{\neg Q}\ ] \vee [\boxed{\text{Q}} \wedge \neg R]] \vee [\neg P \vee \boxed{\text{R}}\ ]$

(c) $[P \wedge Q] \to R \Leftrightarrow \neg[\boxed{\text{P}}\ \boxed{\wedge}\ \boxed{\text{Q}}\ ] \vee R$
$\Leftrightarrow [\boxed{\neg P}\ \boxed{\vee}\ \neg Q] \vee R$
$\Leftrightarrow \boxed{\neg P}\ \boxed{\vee}\ [\neg Q \vee R]$

$\Leftrightarrow$ $\boxed{\neg P}$ $\boxed{\vee}$ $[Q \to R]$

$\Leftrightarrow$ $\boxed{\text{P}}$ $\to [Q \to R]$

6 (a) Express the argument given below using the symbol suggested for each proposition. [8]
(b) Check whether or not the reasoning is correct using inference rules on the wffs (symbolic form) of (a). **No credit will be given if your reasoning is not in symbolic form**. [12]

Either France was not strong (F) or Napoleon made an error (N). It is not the case that if Napoleon did not make an error then Russia was not a superior power (R). If the army did not fail (A), then France was strong. Hence the army failed and Russia was a superior power.

(a) $F \vee N$
$\neg(\neg N \to R)$
$A \to \neg F$

$\neg A \wedge \neg R$

(b) $\neg(\neg N \to R)$
$\neg(\neg N \to R) \to \neg N \wedge \neg R$

$\neg N \wedge \neg R$
$(\neg N \wedge \neg R) \to \neg N$

$\neg N$
$F \vee N$

F
$A \to \neg F$

$\neg A$

Also
$\neg N \wedge \neg R$
$(\neg N \wedge \neg R) \to \neg R$

$\neg R$

$\neg A$
$\neg R$

$\neg A \wedge \neg R$