CS 381 Test

October 2013

1. Prove that the sum of a rational number and an irrational number is irrational. [10]

Suppose that it is not true, that is, the sum of a rational number m and an irrational number n is rational.

Then there are integers p and q such that

$$m + n = p/q.$$

Since m is rational, there are integers s and t such that $s/t + n = p/q$.

Hence $n = p/q - s/t$.

$$= \frac{pt - sq}{qt}$$

Since p, q, s, t are integers, $pt - sq$ and qt are integers. Hence $\frac{pt-sq}{qt}$ is rational. Hence n is rational, contradicting the hypothesis. Hence the claim must be true.

2. Fill in the blanks with the **SHORTEST** string of characters so that the resultant proposition is valid. [15]

(a) $(P \wedge Q) \rightarrow P \Leftrightarrow \neg(P \wedge Q) \vee \boxed{?\ P}$

$\Leftrightarrow (\boxed{?\ \neg P} \vee \neg Q) \vee P \Leftrightarrow P \vee (\boxed{?\ \neg P} \vee \neg Q)$

$\Leftrightarrow (P \vee \boxed{?\ \neg P}) \vee \neg Q \Leftrightarrow \boxed{?\ \text{True}} \vee \neg Q \Leftrightarrow \boxed{?\ \text{True}}$

(b) $\neg\exists x \exists y[[P(x) \wedge Q(y)] \rightarrow R(x, y)]$

$\Leftrightarrow \forall x \neg \boxed{?\ \exists y}[[P(x) \wedge Q(y)] \rightarrow R(x, y)]$

$\Leftrightarrow \forall x \forall y \neg[[P(x) \wedge Q(y)] \rightarrow R(x, y)]$

$\Leftrightarrow \forall x \forall y[P(x) \wedge Q(y) \wedge \boxed{?\ \neg R(x,y)}]$

$\Leftrightarrow \forall x P(x) \wedge \boxed{?\ \forall x} \forall y[Q(y) \wedge \neg R(x, y)]$

$\Leftrightarrow \forall x P(x) \wedge \forall y Q(y) \wedge \boxed{?\ \forall x \forall y} \neg R(x, y)$

3. Express the assertions given below as propositions of predicate logic using the following predicates. The universe is the set of objects. [15]

$F(x)$: x is a flower.
$L(x, y)$: x likes y.
$P(x)$: x is a person.
$R(x)$: x is red.

a) Some person likes some flower.

$\exists x \exists y\ [P(x) \wedge F(y) \wedge L(x,y)]$ or equivalent.

b) Some person likes a flower only if it is red.

$\exists x \forall y\ [[P(x) \wedge L(x,y) \wedge F(y)] \rightarrow R(y)]$ or equivalent

c) Not every person likes a red flower.

$\neg \forall x \exists y\ [[P(x) \wedge F(y) \wedge R(y)] \rightarrow L(x,y)]$ or equivalent,

4. Restate the following assertions in "if ... then ..." form: [15]

(a) I am happy only if I am free.

If I am happy, then I am free.

(b) $x = 1$ is necessary for $x^2 = 1$.

If $x^2=1$, then $x=1$.

(c) Pythagoras' theorem holds if a triangle is a right triangle.

If a Triangle is a right triangle, then Pythagoras's theorem holds.

5. For the following assertion answer the questions below: [15]

Assertion: If some printer is busy or out of service,
then some printer job is lost.

(a) Find the converse in English.

If some printer job is lost, some printer is busy or out of service.

(b) Find the contrapositive in English.

If no printer job is lost, then no printer is busy or out of service.

(c) Find the negation in English. DO NOT simply say "It is not the case that ..." or something similarly trivial.

Some printer is busy or out of service and (but) no printer job is lost.

6. Indicate which ones of the following are true and which ones are false: [15]

(a) $\emptyset \in \{\{\emptyset\}, \{\{\emptyset\}\}\}$

F

(b) $\{0\} \subseteq 2^{\{0,1\}}$

F

(c) $0 \in \emptyset$

F

(d) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

T

(e) $\{\emptyset\} \subseteq \{0\}$

F

(f) $\emptyset \subseteq \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

T

(g) $\{x\} \in \{x\}$

F

(h) $A \subseteq A \times B$, where A and B are sets.

F

(i) $x \subseteq \{x, \{x\}\}$

F

(j) $\emptyset \in \{0\}$

F

7. Prove that $B \subseteq A$ if $B - A = \emptyset$, where A and B are sets. [15]

Suppose that it is not true, ~~that~~ is, $B-A=\phi$ and $B \nsubseteq A$.

Then since $B \nsubseteq A$, there exists an element x such that $x \in B$ and $x \notin A$.

That means that $x \in B-A$.

Hence $B-A \neq \phi$, contradicting the hypothesis that $B-A=\phi$.

Hence the claim must be true.