CS 600 Final Exam

December 15, 1999

1. Find the big-oh relationships for the following functions. **Give your calculations.** [15]

$$n^{1/3},\ (3/2)^n,\ 2^n,\ (\ln n)^2.$$

2. Let L be an array of size n, let $L[i]$ denote the i-th key of L, let x be the key being searched for in L, and let $p(i)$ be the probability for $x = L[i]$. Suppose that x is always found in L with the following probability:

$$\begin{aligned} p(i) &= c(n/2 - i) && \text{for } 1 \le i \le n/2 \\ &= c(i - n/2) && \text{for } n/2 + 1 \le i \le n \end{aligned}$$

where c is a quantity that does not depend on i.

(a) Formulate the equation for computing the average number of comparisons (i.e. average time) needed for the Sequential Search with the probability distribution given above. [10]
(b) Compute the average time from (a). You do not have to compute the value of c. [10]

You may use the following formulas if you need them:
$\Sigma_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2$, $\Sigma_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$,
$\Sigma_{i=1}^{n} i^3 = (n(n+1)/2)^2$, $\lg(n!) = \Theta(n \lg n)$.

3 (a) Use the separable programming technique to formulate an approximate linear programming model for the following problem. Use $x_1, x_2 = 0,\ 1,\ 2,\ 3$ as the breakpoints of the piecewise linear functions. [10]

Max $z = 3.6x_1 + 4x_2 - x_1^2 - x_2^3$
Subject to

$$\begin{aligned} &-x_1 - x_2 \le -1 \\ &-x_1 + x_2 \le 1 \\ &x_1 \le 2 \\ &x_1 \ge 0,\ x_2 \ge 0. \end{aligned}$$

(b) Show that if the slack varaibles are taken as the initial basic variables for the approximation problem, then the basic solution is infeasible. [5]

(c) Formulate the auxiliary problem for finding an initial basic feasible solution for the approximation problem. [5]

4. Solve the following knapsack problem by branch-and-bound. [15]

Max $z = 5x_a + 3x_b + 5x_c + 4x_d$
Subject to

$$31x_a + 16x_b + 32x_c + 23x_d \leq 75$$

Note that the variables are **NOT** ordered in the right order.

5. Solve the following "Scheduling on Uniform Parallel Machines" problem by using the labeling algorithm. [15]

A set of 4 jobs on 3 uniform parallel machines must be scheduled.
The number of machine-days p_i required to complete the job, a release date r_i (representing the beginning of the day when job i becomes available for processing), and a due date $d_i \geq r_i + p_i$ (representing the beginning of the day by which the job must be completed) are given in the table below.
We assume that a machine can work on only one job at a time and that each job can be processed by at most one machine at a time. The processing of a job can be interrupted and it can be given to different machines on different days.

Job i	1	2	3	4
Processing time p_i	2.1	1.5	3.6	1.25
Release time r_i	3	3	5	1
Due date d_i	7	5	9	4

6. Consider the following job scheduling problem: There are n jobs, J_i, $i = 1, 2, ..., n$, and Job J_i requires p_i machine-days to process, Also there are k machines of the same kind. Any machine can be used to process a job but once a job is started on a machine, it can not be switched to another machine because setting up time is too large. All the jobs must be finished by day D (starting on or after day 1).
Find a schedule of the jobs so that all the jobs are completed by day D.

Prove or disprove that this problem is NP-complete. [15]