CS 381 Solutions to Homework 9

Q 1. Textbook p. 527:
1. (b) $< 1, 3 >, < 2, 2 >, < 3, 1 >, < 4, 0 >$
(c) $< 1, 0 >, < 2, 0 >, < 2, 1 >, < 3, 0 >, < 3, 1 > < 3, 2 >, < 4, 0 >, < 4, 1 >$ $, < 4, 2 >, < 4, 3 >$
(e) $< 0, 1 >, < 1, 0 >, < 1, 1 >, < 1, 2 >, < 1, 3 >, < 2, 1 > < 2, 3 >, < 3, 1 >$ $, < 3, 2 >, < 4, 1 >, < 4, 3 >$

2 (a) $< 1, 1 >, < 1, 2 >, < 1, 3 >, < 1, 4 >, < 1, 5 >, < 1, 6 >, < 2, 2 >, < 2, 4 >, < 2, 6 >, < 3, 3 >, < 3, 6 >, < 4, 4 >, < 5, 5 >, < 6, 6 >$

Q 2. Let R denote the relation to be defined.
Basis Clause: $< 0, 0 > \in R$
Inductive Clause: If $< x, y > \in R$, then $< x + 1, y + 3 > \in R$.
Extremal Clause: Nothiung is in R unless it is obtained from the Basis and Inducive Clauses.

Q3. Every subset of $\{< 1, 1 >, < 1, 2 >, < 2, 1 >, < 2, 2 >\}$, including the empty set, is a binary relation on {1,2}. There are 16 of them altogether.

Q 4. Let A denote the set of cardinarity n. Then a unary relation on A is a set of 1-tuples of elements of A, that is a set of $< i >$'s for elements i of A. Hence the number of binary relations on A is equal to the number of subsets of A. Since a set B has $2^{|B|}$ subsets in general, the number of unary relations on A is equal to 2^n.