CS 381 Solutions to Homework 5

p. 56

48 (a) A is true or false.
If A is true, then
$\forall[A \rightarrow P(x)] \leftrightarrow \forall x[T \rightarrow P(x)] \leftrightarrow \forall x P(x)$
Also $[A \rightarrow \forall x P(x)] \leftrightarrow [T \rightarrow \forall x P(x)] \leftrightarrow \forall x P(x)$
Hence $\forall x[A \rightarrow P(x)] \leftrightarrow [A \rightarrow \forall x P(x)]$.
If A is false, then
$\forall x[A \rightarrow P(x)] \leftrightarrow \forall x[F \rightarrow P(x)] \leftrightarrow T$
Also $[A \rightarrow \forall x P(x)] \leftrightarrow [F \rightarrow \forall x P(x)] \leftrightarrow T$
Hence $\forall x[A \rightarrow P(x)] \leftrightarrow [A \rightarrow \forall x P(x)]$.

p. 68

38 (a) $\exists x \neg M(x)$, where the universe is the set of students in this class and $M(x)$ means x likes mathematics.
Some students in this class donot like mathematics.
(c) $\exists x \exists y \neg T(x, y)$, where x represents a member of the set of students in this class, y represents a math course offered at this school, and $T(x, y)$ means x has taken y.
Every student in this class has not taken some mathematics courses offered at this school.

pp. 79 -80

14 (b) Let the universe be the set of the five roommates.
Also let $D(x)$ mean that x has taken a course in discrete mathematics, and let $A(x)$ mean that x can take a course in algorithms.
Then the premises are $\forall x D(x)$ and $\forall x[D(x) \to A(x)]$.
By universal instantiation to the first premise, $D(c)$ for an arbitrary member c of the universe. Also for that c by universal instantiation of the second premise, $D(c) \to A(c)$. Hence by modus ponens, $A(c)$. Hence by universal generalization, $\forall x A(x)$.
(d) Let the universe be the set of all the students in this class.
Also let $F(x)$ mean that x has been to France, and let $L(x)$ mean that x has visited the Louvre.
Then the premises are $\exists x F(x)$ and $\forall x[F(x) \to L(x)]$.
By existential instantiation to the first premise, $F(c)$ for some c in the universe. Also for that c by universal instantiation to the second premise, $F(c) \to L(c)$. Hence by modus ponens, $L(c)$. Hence by existential generalization, $\exists x L(x)$.

16 (c) Incorrect. Quincy may also like other types of movie.

p. 91

14. If x is rational and $x \neq 0$, then $x = m/n$ for some nonzero integers m and n.
Hence $1/x = n/m$. Hence $1/x$ is rational.

18 (b) Suppose that n is not even.
Then $n = 2k + 1$ for some integer k.
Then $3n + 2 = 3(2k + 1) + 2 = 6k + 5$.
Hence $3n + 2$ is odd, which contradicts the premise.
Hence n must be even.

24. Suppose that it is not true, that is, no three or more days of some 25 chosen days fall into the same month. Then at most two of those 25 chosen days fall into the same month.
Then altogether at most 24 of those 25 days can be found in the 12 months of the year, which can not be true.