CS 381 Solutions to Homework 6

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6. $B \subset A$, $C \subset A$, $C \subset D$

10. (c) is false and the rest are true.

20. (a) 0, (b) 1, (c) 2, (d) 3.

24. (b) and (d)

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16 (e) For an arbitrary x,
$x \in A \cup (B - A) \leftrightarrow x \in A \vee (x \in B \wedge \neg x \in A)$
$\leftrightarrow (x \in A \vee x \in B) \wedge (x \in A \vee \neg x \in A)$
$\leftrightarrow (x \in A \vee x \in B) \wedge True$
$\leftrightarrow (x \in A \vee x \in B)$
$\leftrightarrow x \in A \cup B$
Hence $A \cup (B - A) = A \cup B$

20 (a) For an arbitrary element x, if $x \in B$, then by addition $x \in A \vee x \in B$.
Hence $B \subseteq A \cup B$. — (1)
Also if $x \in A \cup B$, then $x \in A \vee x \in B$.
If $x \in A$, then since $A \subseteq B$ by the hypothesis, $x \in B$.
Hence $A \cup B \subseteq B$. — (2)
Hence by (1) and (2), $A \cup B = B$.

30 (c) Supose that $A \neq B$.
Then without loss of generality we can say that $\exists x (x \in A \wedge \neg x \in B)$.
For that x, $x \in A \cap C$ or $\neg x \in A \cap C$.
Case 1: If $x \in A \cap C$, then $x \in B \cap C$.
Hence $x \in B$ and $x \in C$.
That contradicts the assumption that $\neg x \in B$.

Case 2: If $\neg x \in A \cap C$, then $x \in A \wedge \neg x \in C$.
Hence $\neg x \in C$.
Since $x \in A$, $x \in A \cup C$.
Hence $x \in B \cup C$ by one of the hypotheses, which means that $x \in B$, since $\neg x \in C$.
But that contradcits the assumption that $\neg x \in B$.

Hence from cases 1 and 2 $A = B$.