CS 390 Final Exam

May, 2004

1. Prove that the following languages are non-regular:

(a) $L_1 = \{0^n 1^m : m \text{ and } n \text{ are natural numbers and } m > n \}$. [8]
(b) $L_2 = \{a^n b^m c^m : m \text{ and } n \text{ are natural numbers and } n \geq 1 \}$. [7]

2. Following the Kleene's theorem, construct an $NFA - \Lambda$ that accepts the language represented by the regular expression $(ab + a)^* b$. **DO NOT SIMPLIFY**. [15]

3. Using the basic Turing machines $T_a, T_b, T_R, T_L, T_\Delta, T_{L_\Delta}$ and T_{R_Δ}, construct a Turing machine that accepts (but not decides) the language
$L = \{a^n b^m c^n : m, n \text{ are natural numbers and } m \geq n \}$. [15]

4. Find an NFA that accepts the same language as the following NFA-Λ:

State q	a	b	Λ	State q	a	b	Λ
1	$\emptyset$	$\emptyset$	{2, 9}	6	$\emptyset$	$\emptyset$	{1}
2	$\emptyset$	$\emptyset$	{3, 7}	7	{8}	$\emptyset$	$\emptyset$
3	{4}	$\emptyset$	$\emptyset$	8	$\emptyset$	$\emptyset$	{1}
4	$\emptyset$	$\emptyset$	{5}	9	$\emptyset$	{10}	$\emptyset$
5	$\emptyset$	{6}	$\emptyset$	10	$\emptyset$	$\emptyset$	$\emptyset$

The initial state is state 1 and the accepting state is state 10.

5. Let S and T be sets of states of an $NFA - \Lambda$. Prove by structural induction (general induction) that if $S \subseteq T$, then $\Lambda(S) \subseteq \Lambda(T)$. [15]

6 Which of the following statements are true and which are false ? No proof is needed. [15]

(a) Every PDA (Pushdown Automaton) has two stacks.
(b) Every CFL (Context-Free Language) is accepted by some PDA.
(c) The following grammar is context-free:

$$S \to aTbS$$
$$S \to bTT$$
$$T \to a$$
$$T \to S$$

(d) The following grammar is context-free but not regular:

$$S \to aS$$
$$S \to bS$$
$$S \to \Lambda$$

(e) The following grammar generates the set of trings that ends in a:

$$S \to SS$$
$$S \to bS$$
$$S \to a$$

(f) There are languages that are not accepted by any PDA.
(g) A regular language is not CFL.
(h) Every CFL is accepted by some Turing machine.
(i) There is exactly one non-terminal on the left hand side of any production of any context-free grammar.
(j) Any language accepted by some PDA is not regular.