1. Define $L^*$ for every language $L$ as follows:

Basis Clause: $\Lambda \in L^*$.

Inductive Clause: For every $x$ and $y$, if $x \in L$ and $y \in L^*$, then $xy \in L^*$.

Extremal Clause: Nothing is in $L^*$ unless it is obtained from the above clauses.

With the above definition, prove by Structural Induction that $(S^*)^* \subseteq S^*$. You may use any of the following results: [16]

(a) If $x \in L^*$ then $x \in L^+$.
(b) If $x \in L^*$ and $y \in L^*$, then $xy \in L^*$.
(c) If $x \in L^*$, then $x \in \bigcup_{i=0}^{\infty} L^i$. 


2. Construct a Turing machine that accepts the language \( \{ww^r | w \in \{a, b\}^*\} \) using basic Turing machines. 
Do Not give detailed transitions table or diagram in terms of individual states. [16]
3. $\text{Accept}_\Lambda$ asks whether or not a given Turing machine accepts the string $\Lambda$. Prove that $\text{Accept}_\Lambda$ is unsolvable knowing that Halting Problem is unsolvable. [16]
4. Prove that the language of even length palindromes, i.e. \( \{ww^r \mid w \in \{a, b\}^*\} \), is non-regular using Myhill-Nerode. [16]
5. Given the following grammar, answer the questions below:

\[ S \to aT \mid bT \mid \Lambda \\
T \to aS \mid bS \]

(a) What is the language generated by this grammar? [6]

(b) Parse the string \( aababb \) top-down. Use \( \Rightarrow \) to express your derivation. [5]

(c) Parse the string \( aababb \) bottom-up. Use \( \Rightarrow \) to express your derivation. [5]

6. Indicate which ones of the following statements are true and which ones are false. [20]

(a) \( 00 \) is in the language \( 01^* + 10^* + 1*0 + (0^*1)^* \).

(b) \( (111^*)^* = (11 + 111)^* \).

(c) \( (00^*11^*)^* = \Lambda + 0(0 + 1)^*1 \).

(d) \( 0101 \) is one of the shortest strings that are not in \( (0^* + 1^*)(0^* + 1^*)(0^* + 1^*) \).

(e) Every string in \( (0^* + 1^*)(0^* + 1^*)(0^* + 1^*) \) has at most one substring \( 01 \).

(f) The language of all strings not containing the substring \( 00 \) is \( (01 + 1)^*0 \).

(g) If \( \delta(1, a) = \{1, 2, 3\} \), \( \delta(1, b) = \{3\} \), \( \delta(2, b) = \{4\} \) and \( \delta(3, b) = \emptyset \) for an NFA, then \( \delta(1, ab) = \{2, 3, 4\} \).

(h) \( \{a^n \mid 1 \leq n \leq 100\} \) is not a regular language.

(i) The union of infinitely many regular languages is regular.

(j) In NFA, \( \delta^*(q, a) = \delta(q, a) \) for any symbol \( a \) and any state \( q \).