CS 390 Final Exam

December, 2005

1. Define L^* for every languagte L as follows:

Basis Clause: $\Lambda \in L^*$.
Inductive Clause: For every x and y, if $x \in L$ **and** $y \in L^*$, then $xy \in L^*$.
Extremal Clause: Nothign is in L^* unless it is obtained from the above clauses.

With the above definition, prove by Structural Induction that $(S^*)^* \subseteq S^*$.
You may use any of the following results: [16]

(a) If $x \in L^*$ then $x \in L^+$.
(b) If $x \in L^*$ and $y \in L^*$, then $xy \in L^*$.
(c) If $x \in L^*$, then $x \in \cup_{i=0}^{\infty} L^i$.

2. Construct a Turing machine that accepts the language $\{ww^r \mid w \in \{a, b\}^*\}$ **using basic Turing machines**.

Do Not give detailed transitions table or diagram in terms of individual states. [16]

3. Accept_Λ asks whether or not a given Turing machine accepts the string Λ. Prove that Accept_Λ is unsolvable knowing that Halting Problem is unsolvable. [16]

4. Prove that the language of even length palindromes, i.e. $\{ww^r \mid w \in \{a, b\}^*\}$, is non-regular using Myhill-Nerode. [16]

5. Given the following grammar, answer the questions below:

$$S \to aT \mid bT \mid \Lambda$$
$$T \to aS \mid bS$$

(a) What is the language generated by this grammar ? [6]

(b) Parse the string *aababb* top-down. Use $\Rightarrow$ to express your derivation. [5]

(c) Parse the string *aababb* bottom-up. Use $\Rightarrow$ to express your derivation. [5]

6. Indicate which ones of the following statements are true and which ones are false. [20]

(a) 00 is in the language $01^* + 10^* + 1^*0 + (0^*1)^*$.

(b) $(111^*)^* = (11 + 111)^*$.

(c) $(00^*11^*)^* = \Lambda + 0(0 + 1)^*1$.

(d) 0101 is one of the shortest strings that are not in $(0^*+1^*)(0^*+1^*)(0^*+1^*)$.

(e) Every string in $(0^* + 1^*)(0^* + 1^*)(0^* + 1^*)$ has at most one substring 01.

(f) The language of all strings not containing the substring 00 is $(01 + 1)^*0$.

(g) If $\delta(1, a) = \{1, 2, 3\}$, $\delta(1, b) = \{3\}$, $\delta(2, b) = \{4\}$ and $\delta(3, b) = \emptyset$ for an NFA, then $\delta(1, ab) = \{2, 3, 4\}$.

(h) $\{a^n \mid 1 \leq n \leq 100\}$ is not a regular language.

(i) The union of infinitely many regular languages is regular.

(j) In NFA, $\delta^*(q, a) = \delta(q, a)$ for any symbol a and any state q.