

## CS 390 Solutions to Test 1

October 12, 2005

1. Which of the following statements are true and which are false ? [5 points each]

- (a)  $(L^*)^+ = L^*$  for an arbitrary language  $L$ .
- (b)  $|L_1L_2| = |L_1| + |L_2|$  for languages  $L_1$  and  $L_2$ .
- (c)  $(0^*1 + 1^*0^* + (10)^*1)^* = (0^*1^*)^*$
- (d) The string 101010 is not in the language represented by  $(1 + 01^*0)^*$ .
- (e) Every string of  $L^+$  can be expressed as the concatenation of some strings of  $L$ .
- (f)  $LL^* = L^*L$

All except (b) are true.

2. Prove by General (Structural) Induction that for arbitrary languages  $L_1$  and  $L_2$ , if  $L_1 \subseteq L_2$ , then  $L_1^* \subseteq L_2^*$  [20]

Basis Step: By the definition of Kleene star,  $\Lambda \in L_2^*$ .

Induction Hypothesis:  $x \in L_1^*$  and  $x \in L_2^*$ .

Inductive Step: We need to show that for an arbitrary element 'a' of  $L_1$ ,  $xa \in L_2^*$ .

Since  $L_1 \subseteq L_2$  and  $a \in L_1$ ,  $a \in L_2$ .

Since  $x \in L_2^*$ , by the definition of  $L_2^*$ ,  $xa \in L_2^*$ .

3 (a) Find a string of minimum length in  $\{0, 1\}$  that is **NOT** in the language corresponding to the regular expression  $(1^* + 01^*0)^*01^*$ . [4]

$\Lambda$

(b) Find a string of minimum length in  $\{0, 1\}$  that is **IN** the language corresponding to the regular expression of (a). [4]

0

4. Define the language **RECURSIVELY** which is represented by each of the following regular expressions:

(a)  $1^*0$  [7]

Basis Clause:  $0 \in L$  ;BR; Inductive Clause: If  $x \in L$  then  $1x \in L$ . ;BR;  
Extremal Clause: Nothing is in  $L$  unless it is obtained from the above two clauses.

(b)  $(1 + 010)^*01^*$  [7]

Basis Clause:  $0 \in L$ .

Inductive Clause: If  $x \in L$  then  $1x, x1, 010x \in L$ .

Extremal Clause: Nothing is in  $L$  unless it is obtained from the above two clauses.

5. Simplify the following regular expressions:

(a)  $(01^*0 + 1^*0 + 1^*)^*$  [7]

$(0 + 1)^*$

(b)  $0(0^*0 + 0^*) + 0^*$  [7]

$0^*$

6. Find a regular expression for each of the following languages over the alphabet  $\{0, 1\}$ :

(a) The set of strings with an even number of 0's. [7]

$(1 + 01^*0)^*$

(b) The language  $L$  defined recursively as follows: [7]

Basis Clause:  $0 \in L$

Inductive Clause: If  $x \in L$  then  $1x, 010x, x1 \in L$

Extremal Clause: Nothing is in  $L$  unless it is obtained from the above two clauses.

$(1 + 010)^*01^*$