1. Which of the following statements are true and which are false? [5 points each]

(a) \((L^*)^+ = L^*\) for an arbitrary language \(L\).
(b) \(|L_1 L_2| = |L_1| |L_2|\) for languages \(L_1\) and \(L_2\).
(c) \((0^*1 + 1^*0^* + (10)^*1)^* = (0^*1^*)^*\)
(d) The string 101010 is not in the language represented by \((1 + 01^*0)^*\).
(e) Every string of \(L^+\) is can be expressed as the concatenation of some strings of \(L\).
(f) \(LL^* = L^*L\)

All except (b) are true.

2. Prove by General (Structural) Induction that for arbitrary languages \(L_1\) and \(L_2\), if \(L_1 \subseteq L_2\), then \(L_1^* \subseteq L_2^*\) [20]

Basis Step: By the definition of Kleene star, \(\Lambda \in L_2^*\).
Inductive Hypothesis: \(x \in L_1^*\) and \(x \in L_2^*\).
Inductive Step: We need to show that for an arbitrary element ‘a’ of \(L_1\), \(xa \in L_2^*\).
Since \(L_1 \subseteq L_2\) and \(a \in L_1\), \(a \in L_2\).
Since \(x \in L_2^*\), by the definition of \(L_2^*\), \(xa \in L_2^*\).
3. (a) Find a string of minimum length in \{0, 1\} that is \textbf{NOT} in the language corresponding to the regular expression \((1^* + 01^* 0)*01^*\). [4]

(b) Find a string of minimum length in \{0, 1\} that is \textbf{IN} the language corresponding to the regular expression of (a). [4]

4. Define the language \textbf{RECURSIVELY} which is represented by each of the following regular expressions:

(a) \(1^*0\) [7]
Basis Clause: \(0 \in L\)
Inductive Clause: If \(x \in L\) then \(1x \in L\).
Extremal Clause: Nothing is in L unless it is obtained from the above two clauses.

(b) \((1 + 010)^*01^*\) [7]
Basis Clause: \(0 \in L\).
Inductive Clause: If \(x \in L\) then \(1x, x1, 010x \in L\).
Extremal Clause: Nothing is in L unless it is obtained from the above two clauses.
5. Simplify the following regular expressions:
(a) \((01^*0 + 1^*0 + 1^*)^*\)  \[7\]
\((0 + 1)^*\)

(b) \(0(0^*0 + 0^*) + 0^*\)  \[7\]
\(0^*\)

6. Find a regular expression for each of the following languages over the alphabet \(\{0, 1\}\):
(a) The set of strings with an even number of 0’s.  \[7\]
\((1 + 01^*0)^*\)

(b) The language \(L\) defined recursively as follows:  \[7\]
Basis Clause: 0 \(\in\) \(L\)
Inductive Clause: If \(x \in L\) then \(1x, 010x, x1 \in L\)
Extremal Clause: Nothing is in \(L\) unless it is obtained from the above two clauses.
\((1 + 010)^*01^*\)