CS 390 Final Exam

April, 2005

1. Which ones of the following statements are ture and which ones are false ? [16]

(a) If $L_1 \subseteq L_2$ and L_1 is not regular, then L_2 is not regular.
(b) If L_1 and L_2 are nonregular, then $L_1 \cup L_2$ is nonregular.
(c) $(L^+)^* = L^*$.
(d) $S \subseteq \Lambda(S)$.
(e) $S \rightarrow aSa \mid bSb \mid \Lambda$ generates all palindromes over $\{a, b\}$
(f) $(a+b)^*ab(a+b)^* + b^*a^* = (a+b)^*$
(g) aaa is in the language represented by $ab^* + ba^* + b^*a + (a^*b)^*$.
(h) $\{a^n b^n \mid n \in N\}$ is accepted by a PDA.
(i) The set of all odd-length strings in $\{a, b\}^*$ with middle symbol 'a' is generated by a context-free grammar.
(j) Every (Turing-)acceptable language is (Turing-)decidable.

2. Prove by general induction that $Rev(Rev(x)) = x$ for an arbitrary string x in $\{a, b\}^*$. $Rev(x)$ is defined as follows: [16]

Basis Clause: $Rev(\Lambda) = \Lambda$
Inductive Clause: For any string $x \in \{a, b\}^*$ and any symbol c in $\{a, b\}$, $Rev(xc) = cRev(x)$.

3. Prove that $L = \{0^i 1^j \mid j \text{ is a multiple of i}\}$ is nonregular by Myhill-Nerode. [16]

4. Find an examle of a nonregular language $L \subseteq \{a, b\}^*$ so that L^* is regular. [16]

5. Decide whether or not the following statement is true and give your reason: [16]

If L_1 is regular, L_2 is nonregular and $L_1 \cap L_2$ is regular, then $L_1 \cup L_2$ is nonregular.

6. Using the basic Turing machines $T_a, T_b, T_R, T_L, T_\Delta, T_{L_\Delta}$, T_{R_Δ} etc., construct a Turing machine that copies a given string over the alphabet $\{a, b\}$ i.e. a Turing machine that goes from $(q_0, \underline{\Delta}w)$ to $(h, \underline{\Delta}w\Delta w)$. [20]