CS 390 Test

March 2005

1. Describe as simply as possible the language represented by each of the following regular expressions:

(a) $((a+b)^3)^*(\Lambda + a + b)$

The language of strings of a's and b's of length a multiple of 3 or a multiple of 3 plus 1.

(b) $(aa + aaa)^*$

The language of all strings of a's except a.

2. Simplify each of the following regular expressions:

(a) $(a(a+b)^*)^+$

$a(a+b)^*$

(b) $(a(a^* + b^*) + a^*b^*)^*$

$(a+b)^*$

3. For the language of strings over the alphabet $\{a, b\}$ which have no substring ab answer the following questions:

(a) Give a regular expression corresponding to the language.

b^*a^*

(b) Define the language recursively.

Let L denote the language.
Basis Clause: $\Lambda \in L$.
Inductive Clause: If $x \in L$, then $bx \in L$ and $xa \in L$.
Extremal Clause: Nothing is in L unless it is obtained from the Basis and Inductive Clauses.

4. Find the language accepted by the following NFA:

$ab(bab + abb)^*$

5. Prove that $\Lambda(\Lambda(S)) = \Lambda(S)$ for a set of states S of an NFA-Λ, where $\Lambda(S)$ denotes the Λ-closure of S.

We prove this by proving the following two statements: (1) $\Lambda(S) \subseteq \Lambda(\Lambda(S))$ (2) $\Lambda(\Lambda(S)) \subseteq \Lambda(S)$.

(1) $\Lambda(S) \subseteq \Lambda(\Lambda(S))$: This is true by the definition of Λ-closure.
(2) $\Lambda(\Lambda(S)) \subseteq \Lambda(S)$: This is proven by general induction.
Basis Step: The basis (the set of seeds) of $\Lambda(\Lambda(S))$ is $\Lambda(S)$, which is certainly a subset of $\Lambda(S)$.
Inductive Step:
Induction Hypothesis: For an arbitrary $x \in \Lambda(\Lambda(S))$, $x \in \Lambda(S)$ holds.
We are going to show that all the children of x are in $\Lambda(S)$.
The children of x are in $\delta(x, \Lambda)$. But since $x \in \Lambda(S)$ holds, by the definition of Λ-closure, $\delta(x, \Lambda) \subseteq \Lambda(S)$.
Thus we have proven that the property of being in $\Lambda(S)$ is inherited from one generation to the next generation.
Hence $\Lambda(\Lambda(S)) \subseteq \Lambda(S)$.
Hence from (1) and (2), $\Lambda(\Lambda(S)) = \Lambda(S)$ holds.

6. Find an NFA equivalent to the following NFA-Λ.

State q	a	b	State q	a	b
1	$\{3, 4, 5, 6\}$	{ 5 }	4	$\{4, 5, 6\}$	{ 5 }
2	{ 3 }		5	$\{6\}$	{ 5 }
3	$\emptyset$	{ 2 }	6		

The accepting states are 1, 2 and 6