1. Describe as simply as possible the language represented by each of the following regular expressions:

(a) \(((a + b)^3)^* (\Lambda + a + b)\)

The language of strings of a’s and b’s of length a multiple of 3 or a multiple of 3 plus 1.

(b) \((aa + aaa)^*\)

The language of all strings of a’s except a.

2. Simplify each of the following regular expressions:

(a) \((a(a + b)^*)^+\)

\(a(a + b)^*\)

(b) \((a(a^* + b^*) + a^*b^*)^*\)

\((a + b)^*\)

3. For the language of strings over the alphabet \(\{a, b\}\) which have no sub-string \(ab\) answer the following questions:

(a) Give a regular expression corresponding to the language.

\(b^*a^*\)

(b) Define the language recursively.

Let \(L\) denote the language.

Basis Clause: \(\Lambda \in L\).

Inductive Clause: If \(x \in L\), then \(bx \in L\) and \(xa \in L\).

Extremal Clause: Nothing is in \(L\) unless it is obtained from the Basis and Inductive Clauses.

4. Find the language accepted by the following NFA:
5. Prove that \( \Lambda(\Lambda(S)) = \Lambda(S) \) for a set of states \( S \) of an NFA-\( \Lambda \), where \( \Lambda(S) \) denotes the \( \Lambda \)-closure of \( S \).

We prove this by proving the following two statements: 

1. \( \Lambda(S) \subseteq \Lambda(\Lambda(S)) \)
2. \( \Lambda(\Lambda(S)) \subseteq \Lambda(S) \).

(1) \( \Lambda(S) \subseteq \Lambda(\Lambda(S)) \): This is true by the definition of \( \Lambda \)-closure.

(2) \( \Lambda(\Lambda(S)) \subseteq \Lambda(S) \): This is proven by general induction.

Basis Step: The basis (the set of seeds) of \( \Lambda(\Lambda(S)) \) is \( \Lambda(S) \), which is certainly a subset of \( \Lambda(S) \).

Inductive Step:

Induction Hypothesis: For an arbitrary \( x \in \Lambda(\Lambda(S)) \), \( x \in \Lambda(S) \) holds.

We are going to show that all the children of \( x \) are in \( \Lambda(S) \).

The children of \( x \) are in \( \delta(x, \Lambda) \). But since \( x \in \Lambda(S) \) holds, by the definition of \( \Lambda \)-closure, \( \delta(x, \Lambda) \subseteq \Lambda(S) \).

Thus we have proven that the property of being in \( \Lambda(S) \) is inherited from one generation to the next generation.

Hence \( \Lambda(\Lambda(S)) \subseteq \Lambda(S) \).

Hence from (1) and (2), \( \Lambda(\Lambda(S)) = \Lambda(S) \) holds.

6. Find an NFA equivalent to the following NFA-\( \Lambda \).

<table>
<thead>
<tr>
<th>State ( q )</th>
<th>( a )</th>
<th>( b )</th>
<th>State ( q )</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{3, 4, 5, 6}</td>
<td>{5}</td>
<td>4</td>
<td>{4, 5, 6}</td>
<td>{5}</td>
</tr>
<tr>
<td>2</td>
<td>{3}</td>
<td></td>
<td>5</td>
<td>{6}</td>
<td>{5}</td>
</tr>
<tr>
<td>3</td>
<td>\emptyset</td>
<td>{2}</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The accepting states are 1, 2 and 6.