CS 390 Test

July 15, 2006

1. Which of the following statements are true and which are false ? [4 points each]

(a) $LL^* = L^*L$ **True**
(b) $(L^*)^+ = L^*$ for an arbitrary language L. **True**
(c) Every string of L^+ is can be expressed as the concatenation of some strings of L. **True**
(d) $\mid L_1L_2 \mid = \mid L_1 \mid\mid L_2 \mid$ for languages L_1 and L_2. **False**
(e) $(0^*1 + 1^*0^* + (10)^*1)^* = (0^*1^*)^*$ **True**

2. Prove by **General (Structural) Induction** that for arbitrary languages L, $L^* \subseteq (L^*)^*$ [12]

Note that for any language S, S^* is defined recursively as follows:

Basis Clause: $\Lambda \in S^*$
Inductive Clause: If $z \in S^*$, and $x \in S$, then $xz \in S^*$
Extremal Clause: Nothing is in S^* unless it is obtained from the above two clauses.

Proof
Basis Step: Prove $\Lambda \in (L^*)^*$.
By the definition of S^* for any language S, $\Lambda \in (L^*)^*$.

Induction Hypothesis: For an arbitrary $x \in L^*$, $x \in (L^*)^*$.

Inductive Step: Prove that for every $y \in L$, $yx \in (L^*)^*$ for the above x.
To prove that it is sufficient to show that $y \in L^*$.
Since $\Lambda \in L^*$ by the definition of S^* for any language S, if $y \in L$, then $y\Lambda \in L^*$. Hence $y \in L^*$.

Since $y \in L^*$ and $x \in (L^*)^*$, $yx \in (L^*)^*$.

3. For each of the following regular expressions (1) and (2), answer the questions (a) - (d) below:

Regular expressions:

(1) $(b(a^* + b^*) + b^*a^*)^*$
(2) $a^*b^*a^* + a^*b^* + ba^*$

(a) Find a string of minimum length in $\{a, b\}^*$ that is **NOT** in the language corresponding to the regular expression. [6]

Answer:
(1) None because the regular expression represents the set of all strings.
(2) bab

(b) Find a string of minimum length in $\{a, b\}^*$ that is **IN** the language corresponding to the regular expression. [6]

Answer:
(1) Λ
(2) Λ

(c) Simplify the regular expression. [10]

Answer:
(1) $(a + b)^*$ or anything equivalent to that.
(2) $a^*b^*a^*$ or anything equivalent to that.

(d) Describe in English as simply as possible the language represented by the regular expression. [10]

Answer:
(1) The set of strings consisting of all combinations of any number of a's and b's.
(2) The set of strings that have no a's between b's.

4. Define each of the following languages **RECURSIVELY**:

(a) The set of all strings in $\{a, b\}^*$ that contain the substring aa. [12]

Answer:
Let L_1 denote this language.

Basis Clause: $aa \in L_1$
Inductive Clause: For every $x \in L_1$, ax, bx, xa and xb are in L_1.
Extremal Clause: Nothing is in L_1 unless it is obtained by the above two clauses.

(b) The set of all strings in $\{a, b\}^*$ that start with a and that have no substring aa. [12]

Answer:
Let L_2 denote this language.

Basis Clause: $a \in L_2$
Inductive Clause: For every $x \in L_2$, xb and xba are in L_2.
Extremal Clause: Nothing is in L_2 unless it is obtained by the above two clauses.

5. Find a regular expression for the language accepted by the following NFA: [12]

State q	a	b	State q	a	b
1	$\{2\}$	$\emptyset$	4	$\{2\}$	$\{5\}$
2	$\emptyset$	$\{3\}$	5	$\emptyset$	$\emptyset$
3	$\{4\}$	$\{1\}$			

The initial state is 1 and the accepting state is 5.

Answer:
$a(baa + bba)^*bab$ or anything equivalent to that.