CS 390 Test

April, 2007

1. Which of the following statements are true and which are false ? [2 points each]

a) The union of regular languages is not necessarily regular.
b) $\cup_{i=1}^{n}\{a^i\}$ is regular if n is some specific natural number.
c) For any regular language there is at least one FA that recognizes it.
d) Every non-empty regular language is the union of some regular languages.
e) A regular language can not be the union of any nonregular languages.
f) The intersection of regular languages is regular.
g) The union of infinitely many regular languages is regular.
h) Every regular language is accepted by more than one NFA.
i) The Kleene star of any regular language is regular.
j) The concatenation of any languages is regular.

2 a) What kind of strings does the following grammar generate ? Describe them in a non-recursive way in English. [6 pts]

$S \rightarrow XaXaX$
$X \rightarrow aX \mid bX \mid a$

b) Give the shortest string of the language of a). [3 pts]

c) Which of the three strings aababa, aabbbaaa, baabaaba are in the language ? [6 pts]

3. Design (give a transition table) of a PDA that accepts the language $\{a^n b^n \mid n$ is a natural number.$\}$ [15 pts]

4 a) Prove that the language $L_1 = \{0^n 10^{2n} \mid n \text{ is a natural number.}\}$ is nonregular by showing that any two elements of the infinite set $\{0^n \mid n \text{ is a natural number.}\}$ are distinguishable with respect to L_1. [10 pts]

b) Prove by using Myhill-Nerode theorem that the language
$L_2 = \{wa^{|w|} \mid w \in \{a, b\}^*\}$ is nonregular, where $| w |$ is the length of string w. [10 pts]

5. Prove by general induction that for any strings x and y in $\{a, b\}^*$ $Rev(xy) = Rev(y)Rev(x)$, where Rev denotes the reversal. [15 pts]

You may use the following definitions:

Definition of $\{a, b\}^*$:
Basis Clause: $\Lambda \in \{a, b\}^*$
Inductive Clause: If $x \in \{a, b\}^*$, then $xa \in \{a, b\}^*$ and $xb \in \{a, b\}^*$.
Extremal Clause: Nothing is in $\{a, b\}^*$ unless it is obtained from the above two clauses.

Definition of Rev:
Basis Clause: $Rev(\Lambda) = \Lambda$
Inductive Clause: For any string $x \in \{a, b\}^*$ and any symbol c in $\{a, b\}$, $Rev(xc) = cRev(x)$.

6. Design a Turing machine that accepts the language $L = \{a^m b^n c^{m+n} \mid m, n$ are natural numbers.$\}$ You may use any of the basic Turing machines such as T_a, T_R etc. [15 pts]