CS 390 Solutions to Test

February, 2007

1 (a)Define regular languages over alphabet {a, b} in general terms NON-RECURSIVELY.
Hint: You may want to describe how they are generated. [5]

A regular language is a language obtained from any of $\{\Lambda\}$, $\{a\}$ and $\{b\}$ by applying concatenation, union and Kleene star operations.

(b) Explain in general terms (NOT by examples) relationships among regular languages, regular expressions and finite automata. [5]

A regular language can be represented by regular expressions and accepted by finite automata. Conversely the language accepted by a finite automaton is a regular language and the language represented by a regular expression is a regular language.

(c) Explain in general terms (NOT by examples) what "a language is accepted by a finite automaton" means. [5]

A language is accpted by a finite automaton if and only if all the strings in the language are accepted by the finite automaton. A string is accepted by a finite automaton if the finite automaton ends in one of its accepting states after reading the string.

2. Prove by **General (Structural) Induction** that for any arbitrary language L and for any arbitary strings x and y of L^*, $xy \in L^*$. Do induction on x fixing y. [20]

Note that for any language S, S^* is defined recursively as follows:

Basis Clause: $\Lambda \in S^*$
Inductive Clause: If $z \in S^*$, and $x \in S$, then $xz \in S^*$

Extremal Clause: Nothing is in S^* unless it is obtained from the above two clauses.

We prove the statement for an arbitrarily fixed y by induction on x.

Basis Step: The case when $x = \Lambda$.

Since $xy = \Lambda y = y$ and $y \in L^*$, $xy \in L^*$.

Inductive Step: Assume that for an arbitrary x in L^*, $xy \in L^*$ holds.
Note that x is a parent generation. We now prove the statement for the children of x.
The children of x (in L^*) are zx for every z in L.
So we prove here that $(zx)y \in L^*$ for any arbitrary z in L.

Note that $(zx)y = z(xy)$. Also $xy \in L^*$ by the induction hypothesis that is since we are assuming that it is true for the parent x.
Furthermore $z \in L$. Hence by the definition of L^*, $z(xy) \in L^*$.
Hence $(zx)y \in L^*$.

3. For each of the following regular expressions (1) and (2), answer the questions (a) - (d) below:

Regular expressions:

(1) $(a^*b + ab^* + b^*(a^* + ba^*))^*$
(2) $a^*b^*a^* + a^*b^*a + ba^*$

(a) Find a string of minimum length in $\{a, b\}^*$ that is **NOT** in the language corresponding to the regular expression. [6]

(1) None
(2) bab

(b) Find a string of minimum length in $\{a, b\}^*$ that is **IN** the language corresponding to the regular expression. [6]

(1) Λ
(2) Λ

(c) Simplify the regular expression. [10]

(1) $(a + b)^*$
(2) $a^*b^*a^*$

(d) Describe in English as simply as possible the language represented by the regular expression. [8]

(1) The set of strings consisting of a's and b's including Λ.
(2) The set of strings of the form $a^kb^ma^n$, where k, m and n are natural numbers. (or the set of strings that start with zero or more a's followed by

zero or more b's followed by zero or more a's).

4. For each of the following languages find a regular expression which represents it:

(a) The set of all strings in $\{a, b\}^*$ that contain the substring ab. [10]

$(a + b)^* ab(a + b)^*$

(b) The set of all strings in $\{a, b\}^*$ that do not contain the substring ab. [10]

$b^* a^*$

5. Find a regular expression for the language accepted by the following NFA:

State q	a	b	State q	a	b
1	$\emptyset$	$\{2\}$	4	$\{2, 5\}$	$\emptyset$
2	$\emptyset$	$\{3\}$	5	$\emptyset$	$\emptyset$
3	$\{1\}$	$\{4\}$			

The initial state is 1 and the accepting state is 5. [15]

$b(bab + bba)^* bba$