1. Let \( L \) be an arbitrary language over the alphabet \( \{a, b\} \).
Prove that if \( x \) and \( y \) are strings of \( L^* \), then \( xy \) is also a string of \( L^* \) by
general induction on \( y \) fixing \( x \). Assume that \( L^* \) is defined as follows:

Definition of \( L^* \):
Basis Clause: \( \Lambda \) belongs to \( L^* \).
Inductive Clause: If \( w \) is a string of \( L^* \) and \( x \) is a string of \( L \), then \( wx \) is a
string of \( L^* \).
Extremal Clause: Nothing is in \( L^* \) unless it is obtained by using the above
two clauses. [15 Points]

Proof:
Basis Step: Let \( y = \Lambda \). Then \( xy = x\Lambda = x \in L^* \).
Hence the statement holds for \( y = \Lambda \).
Inductive Step: We assume that \( xy \in L^* \) for \( x, y \in L^* \) and prove that for
any string \( w \) of \( L \), \( x(yw) \in L^* \).
\[ x(yw) = (xy)w. \]
Then since \( xy \in L^* \) and \( w \in L \), by the definition of \( L^* \),
\( (xy)w \in L^* \).
Hence \( x(yw) \in L^* \).

2. Answer the following questions for the regular expression \( a^*b^*a^* + b^*a^*b^* + a^*b^* \): [4 Points Each]

(a) Find a shortest string that is not in the language corresponding to the
given regular expression (the language hereafter).
Answer: abab or baba
(b) Find a shortest string in the language other than \( \Lambda \).
Answer: a or b
(c) Describe the language as simply as possible in English.
Answer: The set of strings of a’s and/or b’s which have at most one sub-
string ab and at most one substring ba.
(d) Simplify the given regular expression.
Answer: \( a^*b^*a^* + b^*a^*b^* \)
(e) Find a regular expression of the reversal of the language.
Answer: \( a^*b^*a^* + b^*a^*b^* \)
3. Find a regular expression for the language accepted by the following NFA:[15 Points]

**Answer:** \(a(a + bab + bba)*ba + (a(a + bab)*bb)\)

4. Find a regular expression for the language defined recursively as

Basis Clause: \(\Lambda\), 01 and 10 belong to \(L\).
Inductive Clause: If \(x\) is a string of \(L\), then \(x01\), \(x10\) and \(11x\) are also strings of \(L\).
Extremal Clause: Nothing is in \(L\) unless it is obtained by using the above two clauses. [15 Points]

**Answer:** \((11)*(\Lambda + 01 + 10)(01 + 10)\) = \((11)*(01 + 10)\)

5. Find the NFA-\(\Lambda\) for the regular expression \((ab)^* + a(ba)^*\) faithfully following the procedure given in the textbook/web notes. Do not simplify.[15 Points]

**Answer:** Omitted

6. Answer the questions below for the following NFA-\(\Lambda\):

(a) Find \(\Lambda\{3\}\) and \(\Lambda\{1, 3\}\) [5 Points]

**Answer:** \(\{1, 2, 3\}\) for both.

(b) Find \(\delta*(2, ba)\). [5 Points]

**Answer:** \(\{1, 2, 4\}\)

(c) Find the NFA that accepts the same language as the given NFA-\(\Lambda\). [10 Points]

**Answer:**

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