CS 390 Test

February 2008

1. Let L be an arbitrary language over the alphabet {a, b}.
Prove that if x and y are strings of L*, then xy is also a string of L* by general induction on y fixing x. Assume that L* is defined as follows:

Definition of L*:
Basis Clause: Λ belongs to L*.
Inductive Clause: If w is a string of L* and x is a string of L, then wx is a string of L*.
Extremal Clause: Nothing is in L* unless it is obtained by using the above two clauses. [15 Points]

Proof:
Basis Step: Let $y = \Lambda$. Then $xy = x\Lambda = x \in L^*$.
Hence the statement holds for $y = \Lambda$.
Inductive Step: We assume that $xy \in L^*$ for $x, y \in L^*$ and prove that for any string w of L, $x(yw) \in L^*$.
$x(yw) = (xy)w$. Then since $xy \in L^*$ and $w \in L$, by the definition of L^*, $(xy)w \in L^*$.
Hence $x(yw) \in L^*$.

2. Answer the following questions for the regular expression a*b*a* + b*a*b* + a*b*: [4 Points Each]

(a) Find a shortest string that is not in the language corresponding to the given regular expression (the language hereafter).

Answer: abab or baba

(b) Find a shortest string in the language other than Λ.

Answer: a or b

(c) Describe the language as simply as possible in English.

Answer: The set of strings of a's and/or b's which have at most one substring ab and at most one substring ba.

(d) Simplify the given regular expression.

Answer: a*b*a* + b*a*b*

(e) Find a regular expression of the reversal of the language.

Answer: a*b*a* + b*a*b*

3. Find a regular expression for the language accepted by the following NFA:[15 Points]

Answer: a(a + bab + bba)*ba + (a(a + bab)*bb)*

4. Find a regular expression for the language defined recursively as

Basis Clause: Λ, 01 and 10 belong to L.
Inductive Clause: If x is a string of L, then x01, x10 and 11x are also strings of L.
Extremal Clause: Nothing is in L unless it is obtained by using the above two clauses. [15 Points]

Answer: $(11)^*(\Lambda + 01 + 10)(01 + 10)^* = (11)^*(01 + 10)^*$

5. Find the NFA-Λ for the regular expression (ab)* + a(ba)* faithfully following the procedure given in the textbook/web notes. Do not simplify.[15 Points]

Answer: Omitted

6. Answer the questions below for the following NFA-Λ:

(a) Find $\Lambda(\{3\})$ and $\Lambda(\{1, 3\})$ [5 Points]

Answer: $\{1, 2, 3\}$ for both.

(b) Find $\delta^*(2, ba)$. [5 Points]

Answer: $\{1, 2, 4\}$

(c) Find the NFA that accepts the same language as the given NFA-Λ. [10 Points]

Answer:

	a	b
1	{1, 2, 4}	{1, 2, 3, 4}
2	{4}	{1, 2, 3}
3	{1, 2, 4}	{1, 2, 3, 4}
4	{1, 2, 3}	$\emptyset$