4 (a) Let L be an arbitrary language over the alphabet {a, b}.
Define L* recursively. [10 Points]

Basis Clause : $\Lambda \in L^*$

Inductive Clause : For every string x and every string y
if $x \in L^*$ and $y \in L$ then $xy \in L^*$.

Extremal Clause : Nothing is in L^* unless it is obtained by the basis and inductive clauses above.

(b)Using your definition of L* prove by general induction (a.k.a. structural induction) that $(L_1 \cap L_2)^* \subseteq L_1^* \cap L_2^*$. [10 Points]

Basis Step : For the basis Λ of $(L_1 \cap L_2)^*$, since $\Lambda \in L_1^*$ and $\Lambda \in L_2^*$, $\Lambda \in L_1^* \cap L_2^*$.

Inductive Step : Suppose for a string $w \in (L_1 \cap L_2)^*$, $w \in L_1^* \cap L_2^*$.
We show that for every string $y \in L_1 \cap L_2$, $wy \in L_1^* \cap L_2^*$.
Since $w \in L_1^* \cap L_2^*$, $w \in L_1^*$ and $w \in L_2^*$.
Also since $y \in L_1 \cap L_2$, $y \in L_1$ and $y \in L_2$.
Hence by the definition of L^*, $wy \in L_1^*$ and $wy \in L_2^*$.
Hence $wy \in L_1^* \cap L_2^*$.