CS 390 Final Exam

May 2010

1. Let L = { $0^i 1^j$: $j = i$ or $j = 3i$, where i and j are natural numbers}. Prove by Myhill-Nerode that L is non-regular. [15]

2. Let S be the language over alphabet $\{a, b\}$ defined recursively as follows:

Basis Clause: $\Lambda \in S$
Inductive Clause: For all x, if $x \in S$, then $ax, axb \in S$
Exrremal Clause: Nothing is in S unlesss it is obtained by the above two clauses.

Show by General Induction that $S \subseteq \{a^i b^j : i \geq j$, and i, j are natural numbers$\}$. [15]

3. Consider the following grammar:

$S \rightarrow TT$
$T \rightarrow aT \mid Ta \mid b$

(a) What kind of grammar is it ? [2]

(b) What is the shortest string in the language generated by the grammar ? [3]

(c) What kind of strings does the grammar generate ? [5]

4 (a) Give a design/plan of a Turing machine that divides a given natural number by 2 and leaves the remainder in the second square from the left end of the tape. [5]

4 (b) Give the diagram of your Turing machine of (a) above. You may use basic Turing machines such as T_a, T_R, T_{R_Δ}, etc. [10]

5. Construct the $NFA - \Lambda$ for the regular expression $a^*b + (ba + b)^*$ using Kleene Theorem 1. Do not simplify your $NFA - \Lambda$. [15]

6. Consider the following two problems, Accepts and Accepts Λ:

Accepts: Given a string x and a Turing machine T, this problem asks whether or not T accepts x.

Accepts Λ: Given a Turing machine T, this problem asks whether or not T accepts Λ.

The following questions are concerned with the unsolvability of those problems.
Let x be a string and T be a Turing machine.

(a) A Turing machine T_1 can be constructed by concatenating T_x with T (first T_x then T), where T_x is a Turing machine that writes x and halts given Λ as an input.
Draw a diagram of T_1 as in 4 (b). [3]

(b) Suppose that T_1 accepts Λ. Does T accept x ? Give your reasons. [6]

(c) Suppose that Accepts is unsolvable. Can we conclude that Accept Λ is unsolvable ? Give your reasons. [6]

7. For each of the following statements answer whether or not it is true. [15]

(a) Union of two context-free languages is context-free.
(b) $\{a^n b^n : 0 \leq n \leq 10^5$, n is an integer $\}$ is regular.
(c) Unin of infinitely many regular languages is regular.
(d) For any arbitrary language L, $\Lambda \in L^+$.
(e) If L_1 and L_2 are non-regular, then $L_1 \cap L_2$ is non-regular.
(f) If L_1 is regular and L_2 is non-regular, then $L_1 \cup L_2$ is non-regular.
(g) $(111^*)^* = (11 + 111)^*$.
(h) For a DFA $\delta^*(q, xy) = \delta^*(\delta^*(q, y), x)$.
(i) $\Lambda(S \cap T) = \Lambda(S) \cap \Lambda(T)$ for sets of states S and T of an NFA-Λ.
(j) If the set of 'yes'-instances of a decision problem is accepted by some Turing machine, then that decision problem is solvable.
(k) If one Turing machine accepts the 'yes'-instances of a decision problem and another (different) Turing machine accepts the 'no'-instances of that decision problem, then that decision problem is solvable.
(l) The set of non-palindromes is non-regular.
(m) $\{a^i b^j c^k : i = j + k,\ i, j, k$ are natural numbers$\}$ is context-free.
(n) $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$.
(o) Let L_1 and L_2 be context-free languages generated by context-free grammars G_1 and G_2, respectively. Let S_1 be the start symbol of G_1 and S_2 be the start symbol of G_2, respectively. Also assume that G_1 and G_2 don't have any non-terminals in common. Then $S \rightarrow S_1 S_2$ together with the productions of G_1 and G_2 generates $L_1 L_2$ and it is context-free.