5 (a) Let L be an arbitrary language over the alphabet {a, b}.
Define L* and L^+ recursively. [5 points]

L^* Basis Clause : $\Lambda \in L^*$

Inductive Clause : If $x \in L^*$ and $y \in L$ for any string x, y, then $xy \in L^*$

Extremal Clause : As usual.

L^+ Basis Clause : $L \subseteq L^+$

Inductive Clause : If $x \in L^+$ and $y \in L$ for any string x, y, then $xy \in L^+$

Extremal Clause : As usual.

(b)Using your definition of L* and L^+ of (a) prove by general induction (a.k.a. structural induction) that L^+ is a subset of L* . [15 points]

Basis Step : To prove $x \in L \rightarrow x \in L^*$.

Proof: Since $\Lambda \in L^*$, by Inductive Clause of definition of L^*, for any $\in L$ $\Lambda x \in L^*$.

$\therefore x \in L^*$.

Inductive Step : To prove that if $x \in L^+$ and $x \in L^*$, then for any $y \in L$, $xy \in L^*$.

Proof : Since $x \in L^*$, by Inductive Clause of definition of L^*, for any $y \in L$

$xy \in L^*$.