6. For sets of states S of an NFA-Λ, prove that Λ(S) is a subset of $U_{p \,\varepsilon\, S}\,\Lambda(\{p\})$, that is Λ(S) is a subset of the union of Λ({p}) for all p ε S. [15 points]

To prove $\Lambda(S) \subseteq \bigcup_{p \in S} \Lambda(\{p\})$

Definition of $\Lambda(S)$

Basis Clause : $S \subseteq \Lambda(S)$

Inductive Clause : If $q \in \Lambda(S)$ for any state q, then $\delta(q, \Lambda) \subseteq \Lambda(S)$.

Proof. Basis Step : To prove $S \subseteq \bigcup_{p \in S} \Lambda(\{p\})$

Let q be an arbitrary state of S. Then $q \in \Lambda(\{q\})$ by definition of $\Lambda(\{q\})$. Hence $q \in \bigcup_{p \in S} \Lambda(\{p\})$

Inductive Step : To prove that if for any $q \in \Lambda(S)$, $q \in \bigcup_{p \in S} \Lambda(\{p\})$ holds, then $\delta(q, \Lambda) \subseteq \bigcup_{p \in S} \Lambda(\{p\})$.

Proof : Since $q \in \bigcup_{p \in S} \Lambda(\{p\})$, there is a state r in S such that $q \in \Lambda(\{r\})$.

By definition of $\Lambda(\{r\})$, if $q \in \Lambda(\{r\})$, then $\delta(r, \Lambda) \subseteq \Lambda(\{r\})$

then $\delta(q, \Lambda) \subseteq \Lambda(\{r\})$

Hence $\delta(q, \Lambda) \subseteq \bigcup_{p \in S} \Lambda(\{p\})$

//