

CS 390 Final Exam

April 2011

1. Prove by *Structural Induction* that $L_1^* \cup L_2^* \subseteq (L_1 \cup L_2)^*$ [12]

2. Construct NFA- Λ for $(a * +(ba)*)b*$ following Kleene Theorem (Part) 1 faithfully. Do not simplify your answer. [12]

3 Prove that $\{ww^r \mid w \in \{a, b\}^*\}$ is nonregular, where w^r is the reversal of w . [12]

4. Given the following grammar, answer the questions below: [8 each]

$$\begin{aligned} S &\rightarrow aT \mid bT \\ T &\rightarrow Sa \mid Sb \mid \Lambda \end{aligned}$$

(a) Generate a string of length 3 using the grammar.

(b) Using the grammar generate a string of length 5 which starts with a and ends in b.

(c) Describe briefly in English the language generated by the grammar.

5. For the following Turing machine answer the questions below: [8 each]

q	σ	$\delta(q, \sigma)$	q	σ	$\delta(q, \sigma)$
q_0	Δ	(q_0, Δ, R)	q_2	I	(q_2, I, R)
q_0	I	(q_0, I, R)	q_2	Δ	(q_3, Δ, L)
q_0	$*$	$(q_1, *, R)$	q_3	I	(q_4, Δ, L)
q_1	I	(q_1, I, L)	q_4	I	(q_4, I, L)
q_1	$*$	(q_2, I, R)	q_4	Δ	(h, Δ, S)
q_1	Δ	(q_5, Δ, L)	q_5	$*$	(q_4, Δ, L)

(a) Describe the operation of the Turing machine using configurations when the input $\Delta I * II\Delta$ is given. [8]

(b) What does the Turing machine do in general ? [8]

6. (a) Give a brief description of a Turing machine which accepts the language L_Y of "yes" instances of the Halting Problem (No need to give a transition diagram or table). [8]

(b) What will happen when a "no" instance of the Halting Problem is given to your Turing machine of (a) ? [8]

(c) Is it possible to modify your Turing machine of (a) so that it decides L_Y ? Why ? [8]