CS 390 Final Exam

April 2012

1. Prove by *Structural Induction* that $d(xy) = d(x) + d(y)$ for strings x and y of $\{a, b\}^*$, where $d(w)$ is the length of string w and it is defined as follows: [12]

Basis Clause: $d(\Lambda) = 0$
Inductive Clause: $d(x\sigma) = d(x) + 1$, where x is a string and σ represents a or b.

2. Convert the following NFA to DFA that accepts the same language: [12]

q	a	b
1	$\{ 2, 3 \}$	$\{ 3 \}$
2	$\{ 4 \}$	$\{ 3 \}$
3	$\{ 2, 4 \}$	$\emptyset$
4	$\emptyset$	$\{ 1 \}$

Here the initial state is 1 and the accepting state is 4.

3. Let us define the language L recursively as follows:

Basis Clause: $\Lambda \in L$
Inductive Clause: For any strings x and y if they are in L, then (x) and xy are also in L.
Extremal Clause: Nothing is in L unless it is obtained by the above two clauses.

(a) Find all the strings of L of length 6. [5]

(b) Prove that L is not regular. [12]

4. Given the following grammar, answer the questions below:

$$S \rightarrow SaS \mid b \mid \Lambda$$

(a) Find all the strings of length 3 using the grammar. [5]

(b) Describe briefly in English the language generated by the grammar. Do not just paraphrase the productions.[10]

5. For the following Turing machine answer the questions below:

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	Δ	(q_1,Δ,R)	q_2	Δ	(q_3,Δ,L)
q_1	a	(q_2,Δ,R)	q_3	b	(q_4,Δ,L)
q_1	Δ	(h,Δ,S)	q_4	b	(q_4,b,L)
q_2	a	(q_2,a,R)	q_4	Δ	(q_1,Δ,R)
q_2	b	(q_2,b,R)	q_4	a	(q_4,a,L)

Here q_0 is the initial state.

(a) Describe the operation of the Turing machine using configurations when the input *abb* is given. [5]

(b) What language does the Turing machine accept ? [10]

(c) Does the Turing machine decides the language of (b) ? [7]

6. Construct a Turing machine that accepts the language $\{a^n b^n c^n \mid n$ is a natural number $\}$. You may use basic machines such as T_R: move the head one position to the right; T_{S_L}: shift the tape contents to the right of the head one position to the left; T_A: write A etc. [12]

7. Briefly but precisely explain what the following statement means: [10]

Statement: The halting problem is unsolvable.