CS 390 Test

March 2012

1. Answer the questions below for the language L defined recursively as follows:

Basis Clause: $a \in L$.
Inductive Clause: For any string x, if $x \in L$, then xaa, xab and $xba \in L$.
Extremal Clause: Nothing is in L unless it is obtained by the above two clauses.

Questions:

(a) Obtain all the strings of L of length 5. [4]

aaaaa aaaab aaaba
aabaa aabab aabba
abaaa abaab ababa

(b) Describe L in English. The simpler the better. [8]

L is the set of strings that start with a, are of odd length and have no substring bbb (or every b is either preceded or followed immediately by a).

(c) Find a regular expression for L. [8]

$a(aa+ab+ba)^*$

2. Let S and T be languages over $\{a, b\}$.

(a) Give a recursive definition of $(S \cap T)^*$ following the definition of Kleene star *. [8]

Basis Clause : $\Lambda \in (S \cap T)^*$

Inductive Clause : For all $w \in (S \cap T)^*$ and for all $x \in S \cap T$

$wx \in (S \cap T)^*$.

Extremal Clause : As usual.

(b) Prove by **General Induction (Structural Induction)** that $(S \cap T)^* \subseteq S^* \cap T^*$. [10]

B.S. $\Lambda \subseteq S^* \cap T^*$?

Since $\Lambda \in S^*$, $\Lambda \in T^*$ by the def. of S^* and T^*,

$\Lambda \in S^* \cap T^*$.

I.S: I.H. $w \in (S \cap T)^*$, $w \in S^* \cap T^*$.

$\forall x \in (S \cap T)$. $x \in S$, $x \in T$.

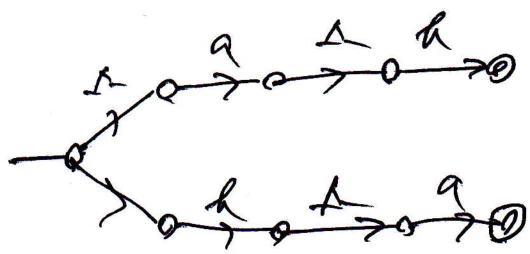
and since $w \in S^*$ and $w \in T^*$

$wx \in S^*$ and $wx \in T^*$. by the def. of S^* and T^*.

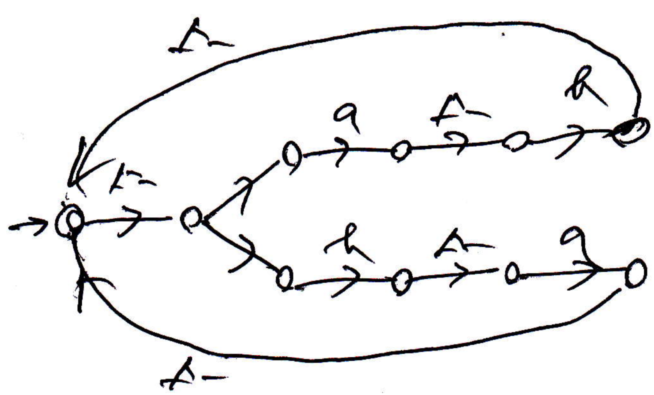
$\therefore wx \in S^* \cap T^*$.

3 (a) Construct an NFA-Λ for $a(ab+ba)^*$ following Part 1 of Kleene Theorem faithfully. Do not simplify your answer. [8]

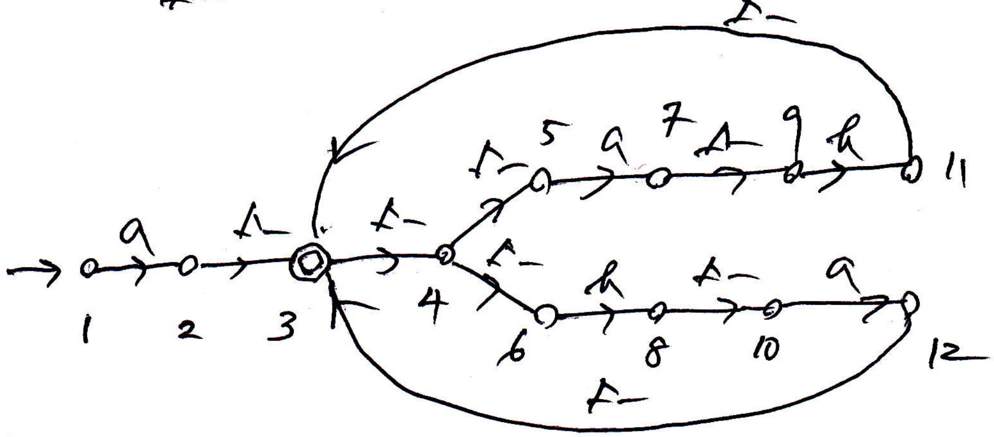
ab   ba



ab+ba



(ab+ba)*



a(ab+ba)*

(b) Convert the NFA-Λ of (a) to an NFA with no Λ-transitions that accepts the same language.[8]

NFA.

	a	b
1	$\{2,3,4,5,6\}$	ϕ
2	$\{7,9\}$	$\{8,10\}$
3	$\{7,9\}$	$\{8,10\}$
4	$\{7,9\}$	$\{8,10\}$
5	$\{7,9\}$	ϕ
6	ϕ	$\{8,10\}$
7	ϕ	$\{3,4,5,6,11\}$
8	$\{3,4,5,6,12\}$	ϕ
9	ϕ	$\{3,4,5,6,11\}$
10	$\{3,4,5,6,12\}$	ϕ
11	$\{7,9\}$	$\{8,10\}$
12	$\{7,9\}$	$\{8,10\}$

4. Simplify the following regular expressions:

(a) $a(a^* + a) + a^*$. [8]

a^*

(b) $(a + (b + ba)^* + baa)^*$. [8]

$(a+b)^*$

5. Let S and T be sets of states of an NFA-Λ. **Prove or disprove** that if $\Lambda(S) \subseteq \Lambda(T)$, then $S \subseteq T$. [10]

Let $S = \{q\}$, $T = \{r\}$ and $\delta(r, \Lambda) = \{q\}$.

Then $\Lambda(S) = \{q\}$ and $\Lambda(T) = \{q, r\}$.

Hence $\Lambda(S) \subseteq \Lambda(T)$

but $S \not\subseteq T$.

6. Which of the following statements are true and which are false ? [20]

(a) For an NFA-Λ, $\delta^*(q, \Lambda) = \{q\}$. $\Lambda(\{q\})$.

F

(b) $(xy)^r = x^r y^r$ for strings x and y, where x^r denotes the reversal of x.

F $y^r x^r$

(c) For an NFA, $\delta^*(q, xa) = \delta(\delta^*(q, x), a)$, where x is a string and a is a symbol.

F. $\bigcup_{p \in \delta^*(q,x)} \delta(p, a)$

(d) A language is regular if and only if it is accepted by some DFA.

T.

(e) $(a + b)^*a(a + b)^*a(a + b)^*$ is a regular expression corresponding to the language of strings with exactly two a's.

at least two

F.

(f) $(a + ab)^*$ corresponds to the language of the strings over $\{a, b\}$ that have no substring bb.

F. Can not start with b in addition, "ba" has no bb but not in the language.

(g) aaababa is a string in the language corresponding to $(a + ab)^*$.

T.

(h) For a set of states S of an NFA-Λ, $\Lambda(\Lambda(S)) = \Lambda(S)$.

T.

(i) For a language L, $(L^*)^+ = (L^+)^*$. $L^* \subseteq (L^+)^* \subseteq L^*$. $(L^*)^+ = L^* \cup L^*L^* \cup \cdots \subseteq L^*$.

T.

(j) A string w is accepted by an NFA if and only if $\delta^*(q_0, w) \subseteq A$, where q_0 is its initial state and A is its set of accepting states.

F. $\delta^*(q_0, w) \cap A \neq \phi$.