CS 390Test

March 2013

1(a) Recursively define the language represented by the regular expression a^*b^*. [5] Let L be the language represented by a^*b^*.

Basis Clause : $\Lambda \in L$

Inductive Clause : If $x \in L$, then $ax \in L$ and $xb \in L$

Extremal Clause : usual clause

(b) List all the strings of length 3 or less of the language of (a). [5]

Λ, a, b, aa, ab, bb, aaa, aab, abb, bbb

(c) Describe the strings of the language of (a) in English. [5]

Any number of a's followed by any number of b's

2. Find a regular expression for each of the languages given below:

(a) The language of strings consisting of exactly two $a's$ and any number of $b's$ over $\{a, b\}$. [6]

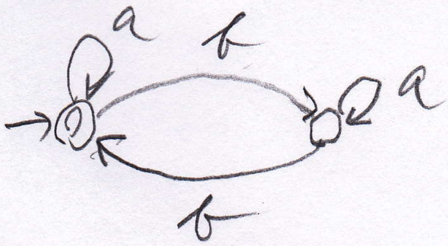
$b^*ab^*ab^*$ or anything equivalent

(b) The language of strings consisting of odd number of $a's$ and any number of $b's$ over $\{a, b\}$. [6]

$(b+ab^*a)^*ab^*$ or $b^*a(b+ab^*a)^*$

or anything equivalent

3. Find an NFA without Λ-transitions that accepts the language represented by the regular expression
$(a + ba^*b)^*$. [8]



Others are possible

4. For the following NFA answer the questions below:

q	a	b
1	$\{2,3\}$	$\{3\}$
2	$\{4\}$	$\{4\}$
3	$\{1,4\}$	$\{2\}$
4	$\emptyset$	$\{4\}$

Here the initial state is 1 and the accepting state is 4.

(a) Find $\delta^*(1, a)$. [5]

$\{2, 3\}$

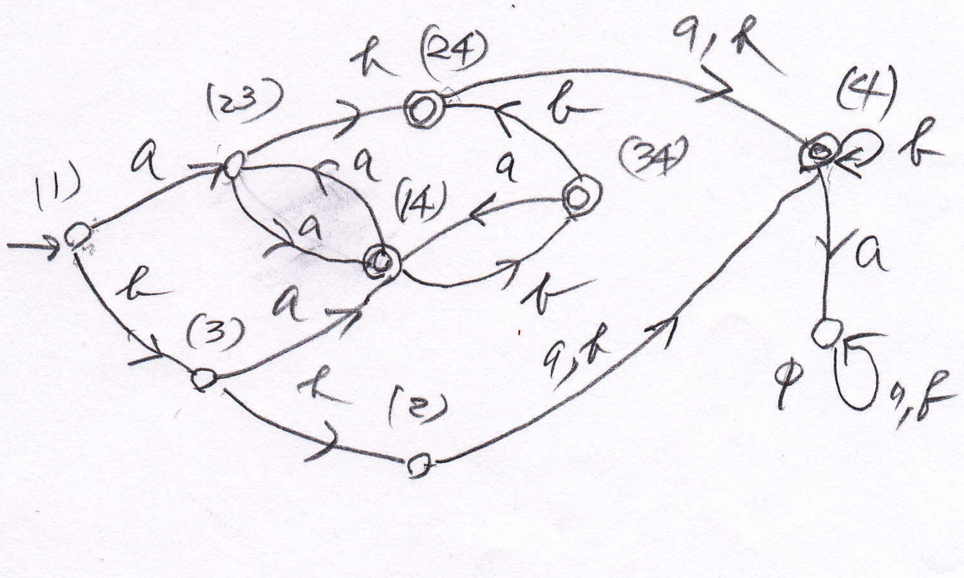
(b) Find $\delta^*(1, ab)$. [5]

$\{2, 4\}$

(c) List all the shortest strings that are accepted by the NFA. [5]

aa, ab, ba

(d) Convert the NFA to a DFA that accepts the same language. [10]



5. For a DFA $(Q, \Sigma, \delta, q_0, A)$, let $h(q, k)$ denote the set of states which can be reached from state q by reading a string of length k. Assuming that $\Sigma = \{a, b\}$, answer the following questions:

(a) Find $h(q, 0)$. [5]

$\{q\}$

(b) Find $h(q, 1)$. [5]

$\{\delta(q, a), \delta(q, b)\}$

(c) Recursively define $h(q, k)$. [5]

Basis Clause: $h(q, 0) = \{q\}$

Inductive Clause:

$$h(q, k+1) = \bigcup_{p \in h(q,k)} \{\delta(p, a)\} \cup \bigcup_{p \in h(q,k)} \{\delta(p, b)\}$$

Extremal Clause: not necessary because k is a natural number.

6. Concerning proving $L^+ \subseteq L^*$ by general induction, answer the following questions:

(a) Briefly explain what needs to be done in general in a proof by general induction. [3]

(1) Prove the claim for the members of the basis.
(2) assuming that the claim holds true for an arbitrary member of the set, prove that it holds true for the children of the member.

(b) What is the basis of L^+ ? [2]

L

(c) What do you need to do in the basis step of a proof by general induction of $L^+ \subseteq L^*$? [3]

Prove that $L \subseteq L^*$

(d) Complete the basis step of the proof. [7]

Since $\Lambda \in L^*$, for any element $x \in L$, $\Lambda x \in L^*$ by the (recursive) definition of L^*.

(e) What do you need to do in the inductive step of a proof by general induction of $L^+ \subseteq L^*$? [3]

assume that for an arbitrary element $x \in L^+$, $x \in L^*$ and prove that $xy \in L^*$ for every $y \in L$.

(f) Complete the inductive step of the proof. [7]

Assume that for an arbitrary element $x \in L^+$, $x \in L^*$.
Then by the (recursive) definition of L^*
if $x \in L^*$ then for every element $y \in L$, $xy \in L^*$.
That is, the children xy of x are in L^*!