CS 600 Final Exam

December 11, 2000

1. Which of the following statements are true ? No justification is required. [20]

(a) If f is o(g) then f is O(g).
(b) The worst case time of my best algorithm to solve the Traveling Salesman Problem is an exponential function of the number of cities. Therefor the Traveling Salesman Problem is NP-complete.
(c) Since the Bin Packing Problem is NP-complete we can not solve it in general.
(d) $n^n = \Theta(2^{2^n})$.
(e) A problem is NP-complete if it can be solved as a known NP-complete problem.
(f) The Minimum Flow Problem with lower bounds is NP-complete.
(g) A Gomory cut reduces the feasible region. It may also exclude integer points if they are not an optimum point.
(h) The worst case time of the simplex method is polynomial in the size of the problem.
(i) The worst case time of my algorithm to solve the Traveling Salesman Problem is smaller for 20 and 30 cities than that for 10 cities. That is because the Traveling Salesman Problem is NP-complete.
(j) If an algorithm can solve a problem by doing backtracking of depth O(ln n) in the worst case, where n is the size of the problem, then the problem is solvable in time polynomial in n in the worst case.

2. Let L be an array of size n, let $L[i]$ denote the i-th key of L, let x be the key being searched for in L, and let $p(i)$ be the probability for $x = L[i]$.
Suppose that x is always found in L with the following probability:

$p(i)$ is twice as likely for $1 \leq i \leq n/4$ as for $n/4+1 \leq i \leq n/2$,
$p(i)$ is twice as likely for $n/4+1 \leq i \leq n/2$ as for $n/2+1 \leq i \leq 3n/4$,
$p(i)$ is twice as likely for $n/2+1 \leq i \leq 3n/4$ as for $3n/4+1 \leq i \leq n$.

Within each of the four intervals, $p(i)$ stays the same (it does not depend on i).

(a) Find the probability $p(i)$ for $3n/4+1 \leq i \leq n$. [5]
(b) Formulate the equation for computing the average time of the Sequential Search with the probability distribution given above. [5]
(c) Compute the average time from (b). [10]

You may use the following formulas if you need them:
$\Sigma_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2$, $\Sigma_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$,
$\Sigma_{i=1}^{n} i^3 = (n(n+1)/2)^2$, $\lg(n!) = \Theta(n \lg n)$.

3. Given the following problem, answer the questions (a) and (b) below:

Max $5x_1 + 6x_2 + 4x_3 + x_4$
Subject to: $51x_1 + 50x_2 + 33x_3 + 11x_4 \leq 163$
$x_1, x_2, x_3, x_4 \geq 0$ and they are integers.

(a) Give a strategy for branching and bounding to solve this problem by Branch-and-Bound. [10]
(b) Solve it using the strategy of (a). [10]

4. Solve the following integer programming problem by using Gomory cut:

Max $x_1 + 2x_2$
Subject to: $-x_1 + 2x_2 \leq 2$
$2x_1 + x_2 \leq 3$
$x_1, x_2 \geq 0$ and they are integers. [20]

IMPORTANT: Use the first inequality (actually the equation derived from it) to generate the cutting plane.

5. k trucks can be used to serve n clients from a single depot. Each client must be visited exactly once. The time for truck k to travel from i to j is denoted by c_{ijk}. The tour of each truck cannot take longer than L_k units of time.
Formulate the problem of finding a shortest total travel time schedule as an integer programming problem. [20]